

Shôichirô Sakai

C^{*}-Algebras
and *W*^{*}-Algebras



Springer-Verlag Berlin Heidelberg New York
1971

Contents

1. General Theory

1.1.	Definitions of C^* -Algebras and W^* -Algebras	1
1.2.	Commutative C^* -Algebras	3
1.3.	Stonean Spaces	6
1.4.	Positive Elements of a C^* -Algebra	7
1.5.	Positive Linear Functionals on a C^* -Algebra	9
1.6.	Extreme Points in the Unit Sphere of a C^* -Algebra	10
1.7.	The Weak Topology on a W^* -Algebra	14
1.8.	Various Topologies on a W^* -Algebra	19
1.9.	Kaplansky's Density Theorem	22
1.10.	Ideals in a W^* -Algebra	24
1.11.	Spectral Resolution of Self-Adjoint Elements in a W^* -Algebra	26
1.12.	The Polar Decomposition of Elements of a W^* -Algebra . .	27
1.13.	Linear Functionals on a W^* -Algebra	28
1.14.	Polar Decomposition of Linear Functionals on a W^* -Algebra	31
1.15.	Concrete C^* -Algebras and W^* -Algebras	33
1.16.	The Representation Theorems for C^* -Algebras and W^* -Algebras	40
1.17.	The Second Dual of a C^* -Algebra	42
1.18.	Commutative W^* -Algebras	45
1.19.	The C^* -Algebra $C(\mathcal{H})$ of all Compact Linear Operators on a Hilbert Space \mathcal{H}	46
1.20.	The Commutation Theorem of von Neumann	48
1.21.	*-Representations of C^* -Algebras, 1	50
1.22.	Tensor Products of C^* -Algebras and W^* -Algebras	58
1.23.	The Inductive Limit and Infinite Tensor Product of C^* -Algebras	70
1.24.	Radon-Nikodym Theorems in W^* -Algebras	75

2. Classification of W^* -Algebras

2.1.	Equivalence of Projections and the Comparability Theorem .	79
2.2.	Classification of W^* -Algebras	83

2.3.	Type I W^* -Algebras	87
2.4.	Finite W^* -Algebras	89
2.5.	Traces and Criterions of Types	95
2.6.	Types of Tensor Products of W^* -Algebras	98
2.7.	*-Representations of C^* -Algebras and W^* -Algebras, 2	102
2.8.	The Commutation Theorem of Tensor Products	108
2.9.	Spatial Isomorphisms of W^* -Algebras	111
 3. Decomposition Theory		
3.1.	Decompositions of States (Non-Separable Cases)	121
3.2.	Reduction Theory (Space-Free)	131
3.3.	Direct Integral of Hilbert Spaces	137
3.4.	Decomposition of States (Separable Cases)	140
3.5.	Central Decomposition of States (Separable Cases)	146
 4. Special Topics		
4.1.	Derivations and Automorphisms of C^* -Algebras and W^* -Algebras	153
4.2.	Examples of Factors, 1 (General Construction)	171
4.3.	Examples of Factors, 2 (Uncountable Families of Types II_1 , II_∞ and III)	183
4.4.	Examples of Factors, 3 (Other Results and Problems)	202
4.5.	Global W^* -Algebras (Non-Factors)	216
4.6.	Type I C^* -Algebras	219
4.7.	On a Stone-Weierstrass Theorem for C^* -Algebras	236
 Bibliography		243
 Subject Index		251
 List of Symbols		255