Dynamic Optimization

The Calculus of Variations and Optimal Control in Economics and Management

Series Volume 4

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