PRINCIPLES OF ECONOMETRICS

HENRI THEIL

Center for Mathematical Studies in Business and Economics The University of Chicago



A Wiley/Hamilton Publication JOHN WILEY & SONS, Inc.

Santa Barbara/New York London/Sydney/Toronto

CONTENTS

 \hat{r}_{1}

List	of	Tables	xxi
List	of	Figures	xxv
List	of	Assumptions	xxvii
List	of	Theorems	xxix
Abb	revi	ations and Other Technical Notes	xxxi
Intr	odu	ction	1
	0.1	Typology of Economic Relations	1
	0.2	Data and Theoretical Relations	4
1	Ma	thematical Tools: Matrix Algebra	7
	1.1	Matrices, Vectors, and Their Elementary Operations	7
	1.2	Partitioned Matrices	16
	1.3	The Solution of Linear Equation Systems	19
	1.4	Quadratic Forms	21
	1.5	Latent Roots and Characteristic Vectors	24
	1.6	Vector and Matrix Differentiation	30
	1.7	Unconstrained Extremum Problems; Least-Squares Adjust-	
		ment	34
	1.8	Constrained Extremum Problems; Least Squares under	
		Linear Constraints	42
	1.9	Principal Component Analysis	46
			XV

.

ī

.

į

2	Stat	istical Tools: Inference and Distribution Theory	57
	2.1	Univariate Distributions	57
	2.2	Multivariate Distributions	63
	2.3	Conditional Distributions and Stochastic Independence	68
	2.4	The Algebra of Expectations	71
	2.5	The Moment-Generating Function	73
	2.6	Distributions Associated with the Normal	80
	2.7	Point Estimation	85
	2.8	Interval Estimation (Confidence Intervals)	93
	2.9	Hypothesis Testing	96
3	Leas	st Squares and the Standard Linear Model	101
	3.1	The Two-Variable Case	102
	3.2	The Assumptions of the Standard Linear Model	106
	3.3	The Least-Squares Estimation Method	111
	3.4	Best Linear Unbiased Estimation and Prediction	119
	3.5	Point Estimation under the Normality Assumption	126
	3.6	Confidence Intervals and Prediction Intervals under the	
		Normality Assumption	130
	3.7	Hypothesis Testing under the Normality Assumption	137
	3.8	The Multicollinearity Problem	147
	3.9	Limitations of the Standard Linear Model	155
4	Part	tial and Multiple Correlation	163
	4.1	The Coefficient of Multiple Correlation	164
	4.2	The Incremental Contributions of Explanatory Variables	167
	4.3	Partial Correlation Coefficients	171
	4.4	Deviations from Means and Adjusted Correlation Coefficients	175
	4.5	Some Useful Diagrams	182
	4.6	Multiple Correlation and Analysis of Variance	186
	4.7	Regression and Correlation in the Multivariate Normal	
		Model	187
5	The	Statistical Analysis of Disturbances	193
	5.1	The Least-Squares Residual Vector	193
		BLUS Residual Vectors	202
	5.3	Properties of Three Families of Residual Vectors	207
	5.4	Tests against Heteroscedasticity and Autocorrelation of	
		Disturbances	214
	5.5	Tests against Nonlinearity	222
	5.6	Further Results on BLUS Residuals	227

CONTENTS xvii

6	Gen	eralized Least Squares and Linear Constraints	236
	6.1	Aitken's Theorem	237
	6.2	Heteroscedasticity and Weighted Least Squares	244
	6.3	Correlated Disturbances and Autoregressive Transformations	250
	6.4	A Class of Distributed Lag Models	258
	6.5	More Complicated Distributed Lag Models	263
	6.6	The Generalized Inverse of a Matrix	268
	6.7	A Singular Disturbance Covariance Matrix	274
	6.8	Constrained Generalized Least-Squares Estimation and	
		Prediction	282
	6.9	The Normal Distribution Theory of the Constrained Singular Case	290
7	The	Combination of Several Linear Relations	294
	7.1	The Joint Generalized Least-Squares Estimation Technique	295
	7.2	An Extension Formulated in Terms of Kronecker Products	303
	7.3	Linear Constraints on Coefficients of Different Equations	312
	7.4	Further Details on the Joint Estimation Procedure	317
	7.5	The Consumer's Allocation Problem: (1) Demand Equations	
		in Infinitesimal Changes	326
	7.6	The Consumer's Allocation Problem: (2) Demand Equations	
		in Finite Changes	330
	7.7	The Consumer's Allocation Problem: (3) Testing and Estima-	
		tion	335
	7.8	Incomplete Extraneous Information and Mixed Estimation	346
	7.9	Inequality Constraints	353
8	Asyr	nptotic Distribution Theory	357
	8.1	Limits, Probability Limits, and Consistent Estimators	357
	8.2	Limiting Distributions and the Central Limit Theorem	366
	8.3	Convergence in Probability and in Distribution: Applications and Further Extensions	372
	8.4	The Cramér-Rao Inequality and the Information Matrix	384
	8.5	The Asymptotic Distribution of Maximum-Likelihood Esti-	
		mators and of Likelihood Ratios	392
	8.6	Asymptotic Properties of Generalized Least-Squares Esti-	
		mators	398
	8.7	Regressions on Lagged Values of the Dependent Variable	408
	8.8	Asymptotic Properties of Some Distributed Lag Estimators	417
	8.9	Sampling Experiments and Higher-Order Approximations	425

.

,)

9	An 1	Informal Introduction to Simultaneous-Equation Models	429
	9.1	Endogenous and Exogenous Variables in a System of Simul-	
		taneous Equations	429
	9.2	Jointly Dependent and Predetermined Variables in a Dynamic	
		Equation System	432
	9.3	Notation and Assumptions	439
	9.4	The Identification Problem	443
	9.5	The Two-Stage Least-Squares Estimation Method	451
	9.6	Recursive Systems	460
	9.7	The Final Form of an Equation System	463
	9.8	The Klein-Goldberger Model: Description of Structural	
		Equations	468
	9.9	The Klein-Goldberger Model: Miscellaneous Comments	475
10	Stati	istical Inference in Simultaneous-Equation Models	484
	10.1	Two Sets of Assumptions and Some Basic Convergence	
		Results	484
	10.2	Conditions for Identification	489
	10.3	Asymptotic Properties of the Two-Stage Least-Squares	
		Estimator	497
	10.4	Limited-Information Maximum Likelihood and the k-Class	500
	10.5	The Three-Stage Least-Squares Method	508
	10.6	A Numerical Example; Mixed Three-Stage Least-Squares	
		Estimation	515
	10.7	Full-Information Maximum Likelihood	524
	10.8	The Choice of an Estimator for Simultaneous Equations	528
11	Spec	ification and Aggregation Analysis	540
	11 1	How Should a Relation Be Specified?	540
		Specification Analysis	548
		Linear Aggregation of Linear Economic Relations	556
		An Example of Aggregation	562
		The Convergence Approach to the Aggregation Problem	570
		The Consumer's Allocation Problem: (4) The Model in	
		Relative Prices	573
	11.7	The Consumer's Allocation Problem: (5) Aggregation over	
		Consumers	580
	11.8	The Consumer's Allocation Problem: (6) Estimation under	
		Conditions of Block-Independence	588
	11.9	The Consumer's Allocation Problem: (7) Asymptotic Evalua-	
		tion of the Estimation Procedure	596

CONTE	INTS XIX
12 Frontiers of Econometrics	603
12.1 Regression Strategies	603
12.2 Errors in the Variables	607
12.3 Robust and Distribution-Free Procedures	615
12.4 Models with Random Coefficients	622
12.5 Probit Analysis and Logit Analysis	628
12.6 Univariate Informational Measures	636
12.7 The Consumer's Allocation Problem: (8) Informa	
Measures of Goodness of Fit	644
12.8 Multivariate Informational Measures	654
12.9 Bayesian Inference	664
Appendix	674
A. Elements of Consumer Demand Theory	674
B. Derivation of the Limited-Information Maximum-Likel	lihood
Estimator	679
Bibliography	687
Tables	717
The t Distribution and the Normal Distribution	717
The χ^2 Distribution	718
5% and 1% Points for the Distribution of F	720
Lower and Upper Bounds of the 5% Points of the Durbin-W	/atson
Test Statistic	724
Lower and Upper Bounds of the 1% Points of the Durbin-W	
Test Statistic	725
5%, 1%, and .1% Points of the Von Neumann Ratio	726
5%, 1%, and .1% Points of the Modified Von Neumann	Ratio 728
Index	731