

Lecture Notes in Economics and Mathematical Systems

Managing Editors: M. Beckmann and W. Krelle

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Kurt Marti

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Descent Directions and Efficient Solutions in Discretely Distributed Stochastic Programs



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