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Meinhard E. Mayer, Introduction to the Fiber-Bundle Approach to Gauge Theories 1

PART II

Wolfgang Drechsler, Gauge Theory of Strong and Electromagnetic Interactions Formulated on a Fiber Bundle of the Cartan Type 145

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