Contents

Introduction ix
A Few Words About Notations xii

PART ONE. FUNDAMENTAL CONCEPTS IN CONTINUUM MECHANICS

1 Describing the Motion of a System: Geometry and Kinematics 3
   1.1 Deformations 3
   1.2 Motion and Its Observation (Kinematics) 6
   1.3 Description of the Motion of a System: Eulerian and Lagrangian Derivatives 10
   1.4 Velocity Field of a Rigid Body: Helicoidal Vector Fields 12
   1.5 Differentiation of a Volume Integral Depending on a Parameter 17

2 The Fundamental Law of Dynamics 21
   2.1 The Concept of Mass 21
   2.2 Forces 27
   2.3 The Fundamental Law of Dynamics and Its First Consequences 28
   2.4 Application to Systems of Material Points and to Rigid Bodies 31
   2.5 Galilean Frames: The Fundamental Law of Dynamics Expressed in a Non-Galilean Frame 35

3 The Cauchy Stress Tensor – Applications 38
   3.1 Hypotheses on the Cohesion Forces 38
   3.2 The Cauchy Stress Tensor 41
   3.3 General Equations of Motion 44
   3.4 Symmetry of the Stress Tensor 46
4 Real and Virtual Powers 48
  4.1 Study of a System of Material Points 48
  4.2 General Material Systems: Rigidifying Velocities 52
  4.3 Virtual Power of the Cohesion Forces: The General Case 54
  4.4 Real Power: The Kinetic Energy Theorem 58

5 Deformation Tensor, Deformation Rate Tensor, Constitutive Laws 60
  5.1 Further Properties of Deformations 60
  5.2 The Deformation Rate Tensor 65
  5.3 Introduction to Rheology: The Constitutive Laws 67

6 Energy Equations and Shock Equations 77
  6.1 Heat and Energy 77
  6.2 Shocks and the Rankine–Hugoniot Relations 82

PART TWO. PHYSICS OF FLUIDS

7 General Properties of Newtonian Fluids 89
  7.1 General Equations of Fluid Mechanics 89
  7.2 Statics of Fluids 95
  7.3 Remark on the Energy of a Fluid 100

8 Flows of Inviscid Fluids 102
  8.1 General Theorems 102
  8.2 Plane Irrotational Flows 106
  8.3 Transonic Flows 116
  8.4 Linear Acoustics 120

9 Viscous Fluids and Thermohydraulics 122
  9.1 Equations of Viscous Incompressible Fluids 122
  9.2 Simple Flows of Viscous Incompressible Fluids 123
  9.3 Thermohydraulics 129
  9.4 Equations in Nondimensional Form: Similarities 131
  9.5 Notions of Stability and Turbulence 133
  9.6 Notion of Boundary Layer 137

10 Magnetohydrodynamics and Inertial Confinement of Plasmas 141
  10.1 The Maxwell Equations and Electromagnetism 142
  10.2 Magnetohydrodynamics 146
  10.3 The Tokamak Machine 148

11 Combustion 153
  11.1 Equations for Mixtures of Fluids 153
  11.2 Equations of Chemical Kinetics 155
Contents

11.3 The Equations of Combustion 157
11.4 Stefan–Maxwell Equations 159
11.5 A Simplified Problem: The Two-Species Model 162

12 Equations of the Atmosphere and of the Ocean 164
12.1 Preliminaries 165
12.2 Primitive Equations of the Atmosphere 167
12.3 Primitive Equations of the Ocean 171
12.4 Chemistry of the Atmosphere and the Ocean 172
Appendix: The Differential Operators in Spherical Coordinates 174

PART THREE. SOLID MECHANICS

13 The General Equations of Linear Elasticity 179
13.1 Back to the Stress–Strain Law of Linear Elasticity: The Elasticity Coefficients of a Material 179
13.2 Boundary Value Problems in Linear Elasticity: The Linearization Principle 181
13.3 Other Equations 186
13.4 The Limit of Elasticity Criteria 189

14 Classical Problems of Elastostatics 191
14.1 Longitudinal Traction–Compression of a Cylindrical Bar 191
14.2 Uniform Compression of an Arbitrary Body 194
14.3 Equilibrium of a Spherical Container Subjected to External and Internal Pressures 195
14.4 Deformation of a Vertical Cylindrical Body Under the Action of Its Weight 198
14.5 Simple Bending of a Cylindrical Beam 201
14.6 Torsion of Cylindrical Shafts 205
14.7 The Saint-Venant Principle 208

15 Energy Theorems – Duality: Variational Formulations 210
15.1 Elastic Energy of a Material 210
15.2 Duality – Generalization 212
15.3 The Energy Theorems 215
15.4 Variational Formulations 218
15.5 Virtual Power Theorem and Variational Formulations 221

16 Introduction to Nonlinear Constitutive Laws and to Homogenization 223
16.1 Nonlinear Constitutive Laws (Nonlinear Elasticity) 224
16.2 Nonlinear Elasticity with a Threshold (Henky’s Elastoplastic Model) 226
Contents

16.3 Nonconvex Energy Functions 228
16.4 Composite Materials: The Problem of Homogenization 230

PART FOUR. INTRODUCTION TO WAVE PHENOMENA

17 Linear Wave Equations in Mechanics 235
17.1 Returning to the Equations of Linear Acoustics and of Linear Elasticity 236
17.2 Solution of the One-Dimensional Wave Equation 239
17.3 Normal Modes 241
17.4 Solution of the Wave Equation 245
17.5 Superposition of Waves, Beats, and Packets of Waves 249

18 The Soliton Equation: The Korteweg–de Vries Equation 252
18.1 Water-Wave Equations 253
18.2 Simplified Form of the Water-Wave Equations 255
18.3 The Korteweg–de Vries Equation 258
18.4 The Soliton Solutions of the KdV Equation 262

19 The Nonlinear Schrödinger Equation 264
19.1 Maxwell Equations for Polarized Media 265
19.2 Equations of the Electric Field: The Linear Case 267
19.3 General Case 270
19.4 The Nonlinear Schrödinger Equation 274
19.5 Soliton Solutions of the NLS Equation 277

Appendix The Partial Differential Equations of Mechanics 279

References 281
Index 285