Introduction to Operations Research

Sixth Edition

Frederick S. Hillier

Professor of Operations Research Stanford University

Gerald J. Lieberman

Professor Emeritus of Operations Research and Statistics Stanford University

TECHNISCHE	HOCHSCHULE Fachbereich 1	DARMSTADT
Gesortrubliothek		
Bei .	winschon	stehre
hive to for. Abstritter	48.267 A 14 / 1	576
sucigoolete		

McGraw-Hill, Inc.

New York St. Louis San Francisco Caracas Auckland Bogotá Madrid Lisbon London Mexico City Milan Montreal New Delhi Singapore Sydney Tokyo Toronto San Juan

Contents

Preface xv

1 INTRODUCTION 1

- 1.1 The Origins of Operations Research 1
- 1.2 The Nature of Operations Research 3
- 1.3 The Impact of Operations Research 4
- 1.4 Algorithms and OR Courseware 4

2 OVERVIEW OF THE OPERATIONS RESEARCH MODELING APPROACH 8

- 2.1 Defining the Problem and Gathering Data 9
- 2.2 Formulating a Mathematical Model 11
- 2.3 Deriving Solutions from the Model 15
- 2.4 Testing the Model 17
- 2.5 Preparing to Apply the Model 19
- 2.6 Implementation 21
- 2.7 Conclusions 22

3 INTRODUCTION TO LINEAR PROGRAMMING 25

- 3.1 Prototype Example 26
- 3.2 The Linear Programming Model 32
- 3.3 Assumptions of Linear Programming 38
- 3.4 Additional Examples 45
- 3.5 Some Case Studies 62
- 3.6 Conclusions 67

4 SOLVING LINEAR PROGRAMMING PROBLEMS: THE SIMPLEX METHOD 81

- 4.1 The Essence of the Simplex Method 82
- 4.2 Setting Up the Simplex Method 87
- 4.3 The Algebra of the Simplex Method 90
- 4.4 The Simplex Method in Tabular Form 95
- 4.5 Tie Breaking in the Simplex Method 100
- 4.6 Adapting to Other Model Forms 103
- 4.7 Post-Optimality Analysis 122

Contents

Х

- 4.8 Computer Implementation 129
- 4.9 The Interior-Point Approach to Solving Linear Programming Problems 131
- 4.10 Conclusions 136

5 THE THEORY OF THE SIMPLEX METHOD 153

- 5.1 Foundations of the Simplex Method 153
- 5.2 The Revised Simplex Method 165
- 5.3 A Fundamental Insight 174
- 5.4 Conclusions 182

6 DUALITY THEORY AND SENSITIVITY ANALYSIS 196

- 6.1 The Essence of Duality Theory 197
- 6.2 Economic Interpretation of Duality 204
- 6.3 Primal-Dual Relationships 206
- 6.4 Adapting to Other Primal Forms 210
- 6.5 The Role of Duality Theory in Sensitivity Analysis 215
- 6.6 The Essence of Sensitivity Analysis 217
- 6.7 Applying Sensitivity Analysis 223
- 6.8 Conclusions 238

7 OTHER ALGORITHMS FOR LINEAR PROGRAMMING 263

- 7.1 The Dual Simplex Method 264
- 7.2 Parametric Linear Programming 266
- 7.3 The Upper Bound Technique 271
- 7.4 An Interior-Point Algorithm 274
- 7.5 Linear Goal Programming and Its Solution Procedures 285
- 7.6 Conclusions 292

8 THE TRANSPORTATION AND ASSIGNMENT PROBLEMS 303

- 8.1 The Transportation Problem 304
- 8.2 A Streamlined Simplex Method for the Transportation Problem 315
- 8.3 The Assignment Problem 329
- 8.4 Conclusions 338

9 NETWORK ANALYSIS, INCLUDING PERT-CPM 353

- 9.1 Prototype Example 355
- 9.2 The Terminology of Networks 356
- 9.3 The Shortest-Path Problem 359
- 9.4 The Minimum Spanning Tree Problem 362
- 9.5 The Maximum Flow Problem 366
- 9.6 The Minimum Cost Flow Problem 372
- 9.7 The Network Simplex Method 378
- 9.8 Project Planning and Control with PERT-CPM 389
- 9.9 Conclusions 404

10 DYNAMIC PROGRAMMING 424

- 10.1 A Prototype Example for Dynamic Programming 425
- 10.2 Characteristics of Dynamic Programming Problems 430

- 10.3 Deterministic Dynamic Programming 433
- 10.4 Probabilistic Dynamic Programming 453
- 10.5 Conclusions 458

11 GAME THEORY 470

- 11.1 The Formulation of Two-Person, Zero-Sum Games 471
- 11.2 Solving Simple Games—A Prototype Example 472
- 11.3 Games with Mixed Strategies 477
- 11.4 Graphical Solution Procedure 479
- 11.5 Solving by Linear Programming 482
- 11.6 Extensions 485
- 11.7 Conclusions 486

12 INTEGER PROGRAMMING 494

- 12.1 Prototype Example 495
- 12.2 Some Other Formulation Possibilities with Binary Variables 497
- 12.3 Some Formulation Examples 503
- 12.4 Some Perspectives on Solving Integer Programming Problems 511
- 12.5 The Branch-and-Bound Technique and Its Application to Binary Integer Programming 515
- 12.6 A Branch-and-Bound Algorithm for Mixed Integer Programming 527
- 12.7 Recent Developments in Solving BIP Problems 533
- 12.8 Conclusions 541

13 NONLINEAR PROGRAMMING 558

- 13.1 Sample Applications 559
- 13.2 Graphical Illustration of Nonlinear Programming Problems 563
- 13.3 Types of Nonlinear Programming Problems 568
- 13.4 One-Variable Unconstrained Optimization 574
- 13.5 Multivariable Unconstrained Optimization 577
- 13.6 The Karush-Kuhn-Tucker (KKT) Conditions for Constrained Optimization 582
- 13.7 Quadratic Programming 586
- 13.8 Separable Programming 591
- 13.9 Convex Programming 598
- 13.10 Nonconvex Programming 603
- 13.11 Conclusions 606

14 MARKOV CHAINS 628

- 14.1 Stochastic Processes 629
- 14.2 Markov Chains 630
- 14.3 Chapman-Kolmogorov Equations 633
- 14.4 Classification of States of a Markov Chain 635
- 14.5 First Passage Times 638
- 14.6 Long-Run Properties of Markov Chains 640
- 14.7 Absorption States 646
- 14.8 Continuous-Time Markov Chains 648

Contents

15 **QUEUEING THEORY** 661

- 15.1 Prototype Example 662
- 15.2 **Basic Structure of Oueueing Models** 662
- 15.3 Examples of Real Queueing Systems 667
- 15.4 The Role of the Exponential Distribution 668
- 15.5 The Birth-and-Death Process 674
- 679 15.6 Oueueing Models Based on the Birth-and-Death Process
- Queueing Models Involving Nonexponential Distributions 15.7 696
- 15.8 **Priority-Discipline Queueing Models** 704
- 15.9 Queueing Networks 709
- 15.10 Conclusions 713

THE APPLICATION OF QUEUEING THEORY 733 16

- 16.1 Examples 734
- 16.2 735 **Decision Making**
- 16.3 Formulation of Waiting-Cost Functions 738
- 16.4 Decision Models 742
- 16.5 Conclusions 748

17 INVENTORY THEORY 756

- 17.1 Examples 757
- 17.2 Components of Inventory Models 759
- 17.3 **Deterministic Models** 761
- 17.4 Stochastic Models 772
- 17.5 Conclusions 797

18 FORECASTING 806

- 18.1 Judgmental Techniques 807
- 18.2 Time Series 808
- 18.3 Forecasting Procedures for a Constant-Level Model 809
- 18.4 A Forecasting Procedure for a Linear Trend Model 812
- 18.5 A Forecasting Procedure for a Constant Level with Seasonal Effects Model 814
- 18.6 Forecasting Errors 817
- 18.7 **Box-Jenkins Method** 818
- 819 18.8 Linear Regression
- 18.9 Conclusions 826

19 MARKOV DECISION PROCESSES 833

- 19.1 A Prototype Example 834
- 19.2 A Model for Markov Decision Processes 836
- 19.3 Linear Programming and Optimal Policies 839
- 19.4 A Policy Improvement Algorithm for Finding Optimal Policies
- 19.5 **Discounted Cost Criterion** 848
- 19.6 Conclusions 855

DECISION ANALYSIS 864 20

- 20.1 A Prototype Example 865
- 20.2 Decision Making without Experimentation 865

843

20.3 Decision Making with Experimentation 869

- 20.4 Decision Trees 874
- 20.5 Utility Theory 877
- 20.6 Conclusions 883

21 SIMULATION 900

- 21.1 Illustrative Examples 902
- 21.2 Formulating and Implementing a Simulation Model 909

. •

- 21.3 Experimental Design for Simulation 919
- 21.4 Regenerative Method of Statistical Analysis 926
- 21.5 Conclusions 932

APPENDIXES 945

- 1Documentation for the OR Courseware9472Convexity9513Classical Optimization Methods9594Matrices and Matrix Operations962
- 4 Matrices and Matrix Operations 963
- 5 Tables 970

Answers to Selected Problems 972

Indexes

Author Index 982

Subject Index 986