

Lecture Notes in Economics and Mathematical Systems

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FB Mathematik TUD



58323764

Nonlinear Dynamical Economics and Chaotic Motion



Fachbereich Mathematik
Technische Hochschule Darmstadt
Bibliothek

Inv.-Nr. B 22282

Springer-Verlag

Berlin Heidelberg New York London Paris Tokyo Hong Kong

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