Lecture Notes in Economics and Mathematical Systems

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Operations Research

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Linear Multiobjective Programming



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Table of Contents

1.	Introduction	1
1.1	The Origin of the Multiobjective Problem and a Sh or t Historical Review	1
1.2.	Linear Multiobjective Programming	4
1.3.	Comment on Notation	5
	LINEAR MULTIBOJECTIVE PROGRAMMING I.	
2.	Basic Theory and Decomposition of the Parametric Space	8
2.1.	Basic Theory - Linear Case	10
2.2.	Reduction of the Dimensionality of the Parametric Space	14
2.3.	Decomposition of the Parametric Space as a Method to Find Nondominated Extreme Points of X	15
2.4.	Algorithmic Possibilities	29
2. 5.	Discussion of Difficulties connected with the Decomposition Method	40
2.5.1.	Some Numerical Examples of the Difficulties	42
	LINEAR MULTIOBJECTIVE PROGRAMMING II.	
3.	Finding Nondominated Extreme Points - A Second Approach (Multicriteria Simplex Method)	63
3.1.	Basic Theorems	63
3.2.	Methods for Generating Adjacent Extreme Points	80
3.3	Computerized Procedure - An Example	93
3.4.	Computer Analysis	114

VIII

LINEAR MULTIOBJECTIVE PROGRAMMING III.

,

4.	A Method for Generating All Nondominated Solutions of X.	122
4.1.	Some Basic Theorems on Properties of N	122
4.2.	An Algorithm for Generating N from Known ^N ex	126
4.3.	Numerical Examples	136
4.3.1.	An Example of Matrix Reduction	136
4.3.2.	An Example of Nondominance Subroutine	141
5.	Additional Topics and Extensions	146
5.1.	Alternative Approach to Finding N_{ex}	146
5.1.1.	The Concept of Cutting Hyperplane	149
5.1.2.	Nondominance in Lower Dimensions	158
5.2.	Some Notes on Nonlinearity	162
5.3.	A Selection of the Final Solution'	167
5.3.1.	Direct Assessment of Weights	169
5.3.2.	The Ideal Solution	171
5.3.3.	Entropy as a Measure of Importance	176
5.3.4.	A Method of Displaced Ideal	180
Bibliography		183
Appendix:		186
Al. A Note on Elimination of Redundant Constraints		187
A.2. Examples of Output Printouts		197
A.3. The Program Description and FORTRAN Printout		202