ELEMENTS OF ADVANCED QUANTUM THEORY

by J. M. ZIMAN, F.R.S.

Professor of Theoretical Physics in the University of Bristol

CAMBRIDGE

AT THE UNIVERSITY PRESS

1969

Lehrstuhl II
für Theoretische Physik
Technische Hochschule Darmstadt

TP 62

CONTENTS

Preface *page* v

1 Bosons

1.1 The simple harmonic oscillator 1

1.2 Annihilation and creation operators 3

1.3 Coupled oscillators: the linear chain 5

1.4 Three-dimensional lattices and vector fields 9

1.5 The continuum limit 12

1.6 Classical field theory 14

1.7 Second quantization 18

1.8 Klein–Gordon equation 21

1.9 Sources of a field, and interactions between fields 22

1.10 Example: Rayleigh scattering of phonons 24

1.11 Example: Yukawa force 26

1.12 Charged bosons 28

2 Fermions

2.1 Occupation-number representation 32

2.2 Annihilation and creation operators: anticommutation 33

2.3 Second quantization 36

2.4 Scattering: connection with statistical mechanics 39

2.5 Interactions between particles: momentum conservation 41

2.6 Fermion–boson interaction 43

2.7 Holes and antiparticles 48

3 Perturbation theory

3.1 The Brillouin–Wigner series *page* 53

3.2 The Heisenberg representation 56

3.3 Interaction representation 60

3.4 Time–integral expansion series 62

3.5 S-matrix 64

3.6 S-matrix expansion: algebraic theory 67

3.7 Diagrammatic representation 74

3.8 Momentum representation 80

3.9 The physical vacuum 86

3.10 Dyson's equation and renormalization 90

4 Green functions

4.1 The density matrix 94

4.2 Equation of motion of density operator 98

4.3 Ensembles in thermal equilibrium 99

4.4 The Kubo formula 101

4.5 The one-particle Green function 104

4.6 Energy–momentum representation 107

4.7 Evaluation of Green functions 110

4.8 Two-particle Green functions 112

4.9 The hierarchy of Green functions 116

4.10 Time-independent Green functions 117

4.11 Matrix representation of the Green function 120

4.12 Space representation of time-independent Green function 122

4.13 The Born series 124

4.14 The T-matrix 127

4.15 Example: impurity states in a metal 131

5 Some aspects of the many-body problem

5.1 Quantum properties of macroscopic systems *page* 135
5.2 Statistical methods: the Thomas–Fermi approximation 136
5.3 Hartree self-consistent field 138
5.4 The Hartree–Fock method 140
5.5 Diagrammatic interpretation of Hartree–Fock theory 143
5.6 The Brueckner method 146
5.7 The dielectric response function 148
5.8 Spectral representation of dielectric function 150
5.9 Diagrammatic interpretation of dielectric screening 154
5.10 The random phase approximation 158
5.11 The Landau theory of Fermi liquids 162
5.12 The dilute Bose gas 167
5.13 The superconducting state 170

6 Relativistic formulations

6.1 Lorentz invariance 175
6.2 Relativistic electromagnetic theory 177
6.3 The wave equation and gauge invariance 180
6.4 Quantization of relativistic fields 183
6.5 Spinors 187
6.6 The Dirac equation 191
6.7 The Dirac matrices 193
6.8 Quantization of the Dirac field 196
6.9 Interactions between relativistic fields 199
6.10 Relativistic kinematics 203
6.11 The analytic S-matrix 207

7 The algebra of symmetry

7.1 Symmetry operations *page* 213
7.2 Representations 215
7.3 Regular representations of finite groups 219
7.4 The orthogonality theorem 222
7.5 Character and class 225
7.6 Product groups and representations 230
7.7 Translation groups 235
7.8 Continuous groups 237
7.9 The rotation group 241
7.10 Irreducible representations of the rotation group 244
7.11 Spinor representations 247
7.12 $SU(2)$ 249
7.13 $SU(3)$ 254

Index 259