

The Stochastic Finite Element Method

**Basic Perturbation Technique
and Computer Implementation**

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