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# Lecture Notes in Economics and Mathematical Systems

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## Degeneracy Graphs and the Neighbourhood Problem



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# C O N T E N T S

1.	INTRODUCTION	1
2.	THE CONCEPT OF DEGENERACY	4
3.	DEGENERACY GRAPHS	10
3.1	The concept of degeneracy graphs	10
3.2	Properties of degeneracy graphs	17
3.2.1	Properties of degeneracy graphs in case of simple degeneracy	17
3.2.2	Properties of degeneracy graphs in case of multiple degeneracy	21
3.3	Degeneracy tableaux	28
4.	ON THE NUMBER OF NODES OF DEGENERACY GRAPHS	34
4.1	The maximum number of nodes of degeneracy graphs	34
4.2	The density of degeneracy tableaux	35
4.3	The minimum number of nodes of degeneracy graphs	39
4.4	On the existence and uniqueness of $\alpha$ xn-degeneracy graphs	52
4.5	An algorithm for determining the number of nodes of degeneracy graphs	57
5.	A METHOD TO SOLVE THE NEIGHBOURHOOD PROBLEM	61
5.1	Examples of the occurrence of the neighbourhood problem	61
5.1.1	The neighbourhood problem in connection with sensitivity analysis	62
5.1.2	The neighbourhood problem in bottleneck linear programming	64
5.2	Solution of the neighbourhood problem by means of degeneracy graphs	66
5.2.1	On the existence of N-minimal trees of a positive degeneracy graph	71
5.2.1.1	The existence of N-minimal trees in case of simple degeneracy	72
5.2.1.2	The existence of quasi-N-minimal trees in case of multiple degeneracy	74

5.2.2	The N-tree method for solving the neighbourhood problem	78
5.2.2.1	The principle of the N-tree method	79
5.2.2.2	Algorithmic description of the procedure	79
5.2.2.3	The N-tree algorithm in programmable form	82
5.2.2.4	Some explanations with respect to TREE and ALL	84
5.2.3	On the efficiency of the N-tree method	85
5.2.3.1	Comparison of TREE and ALL	85
5.2.3.2	Estimations with respect to the number of nodes of TREE-solutions and N-trees	91
5.2.4	On the application of the N-tree method	98
Appendices		101
A.	Basic concepts of linear programming and of theory of convex polytopes	102
B.	Basic concepts of graph theory	107
C.	On $2 \times n$ -degeneracy graphs	109
D.	Flow-charts	119
References		122
Index of symbols		127
Index of terms		128