

Annette Huber

Mixed Motives and their Realization in Derived Categories



Springer

Contents

| | |
|------------------------|----|
| Acknowledgement | v |
| Introduction | vi |

PART I: GENERALITIES

| | |
|---|-----------|
| 1 Basic Notions | 2 |
| 1.1 Simplicial Varieties | 2 |
| 1.2 Axioms of Realization Categories | 6 |
| 1.3 Realization Functors and Cohomology | 9 |
| 2 Derived Categories of Exact Categories | 10 |
| 2.1 Definition | 10 |
| 2.2 Signs in Multiple Complexes | 14 |
| 3 Filtered Derived Categories | 22 |
| 3.1 Constructions | 22 |
| 3.2 Deriving Functors | 26 |
| 3.3 An R-Category | 27 |
| 4 Gluing of Categories | 28 |
| 4.1 How to Glue | 28 |
| 4.2 Calculation of Ext-Groups for Gluings | 35 |
| 4.3 On 1-Extensions | 41 |
| 5 Godement Resolutions | 45 |
| 5.1 Definition | 45 |
| 5.2 Functoriality | 47 |

PART II: CONSTRUCTION OF REALIZATION FUNCTORS

| | |
|---|-----------|
| 6 Singular Cohomology | 50 |
| 6.1 First Step: Realization of Smooth Compactifications | 50 |
| 6.2 Second Step: Real. of Simpl. Smooth Compactif. | 52 |

| | | |
|-----------|---|-----------|
| 6.3 | Third Step: Realization of Simplicial Varieties | 54 |
| 6.4 | Calculation of Cohomology in the Singular Realization | 55 |
| 6.5 | Odds and Ends | 56 |
| 7 | De Rham Cohomology | 57 |
| 7.1 | First Step | 57 |
| 7.2 | Second Step | 58 |
| 7.3 | Third Step | 59 |
| 7.4 | Calculation of Cohomology in the de Rham Realization | 59 |
| 8 | Hodge Realization | 60 |
| 8.1 | The Category of Hodge Complexes | 60 |
| 8.2 | Comparison Functors | 64 |
| 8.3 | The Realization and Ends of the Proofs | 67 |
| 8.4 | Calculation of Cohomology | 70 |
| 8.5 | Real Variant | 71 |
| 8.6 | Remarks | 72 |
| 9 | l-adic Cohomology | 73 |
| 9.1 | Galois Modules | 73 |
| 9.2 | First Step | 77 |
| 9.3 | Second and Third Step | 78 |
| 9.4 | Calculation of Cohomology | 79 |
| 10 | Comparison Functors: l-adic vs. Singular | 80 |
| 10.1 | Homological Algebra | 81 |
| 10.2 | Construction | 83 |
| 11 | The Mixed Realization | 86 |
| 11.1 | The Category of Mixed Realizations | 86 |
| 11.2 | First, Second and Third Step | 90 |
| 11.3 | Calculation of Cohomology | 92 |
| 11.4 | Remarks | 94 |

PART III: THE BLOCH-OGUS AXIOMS

| | | |
|-----------|--|-----------|
| 12 | The Tate Twist | 98 |
| 12.1 | The Twisted Hodge Realization | 98 |
| 12.2 | The Twisted l-adic Realization | 99 |
| 12.3 | The Twisted Mixed Realization | 101 |

| | | |
|-----------|---|------------|
| 13 | \otimes-Product and Internal Hom on D_{MR} | 102 |
| 13.1 | $D_{\mathcal{H},\sigma}$ as \otimes -R-Category | 102 |
| 13.2 | \otimes and Internal Hom in the l-adic Realization | 104 |
| 13.3 | The Mixed Realization | 107 |
| 13.4 | Tensor Products of Complexes of Sheaves | 111 |
| 14 | The Künneth Morphism | 112 |
| 14.1 | Definition in the Smooth Case | 112 |
| 14.2 | The Alexander Whitney Morphism | 116 |
| 14.3 | Definition in the General Case | 118 |
| 14.4 | Cup-Products | 120 |
| 15 | The Bloch-Ogus Axioms | 122 |
| 15.1 | Cohomology with Support | 122 |
| 15.2 | Cohomology with Compact Support | 124 |
| 15.3 | Homology | 133 |
| 15.4 | Some Further Properties | 137 |

PART IV: CHERN CLASSES

| | | |
|-----------|---|------------|
| 16 | The Chern Class of a Line Bundle | 140 |
| 17 | Classifying Spaces | 146 |
| 17.1 | Operation of Finite Groups | 146 |
| 17.2 | The Splitting Principle | 149 |
| 17.3 | B, Gl and Universal Chern Classes | 151 |
| 17.4 | The Realization Complex of B, Gl | 153 |
| 18 | Higher Chern classes | 154 |
| 18.1 | Higher K-Theory | 154 |
| 18.2 | Definition | 159 |
| 18.3 | Multiplicativity | 164 |

PART V: RELATION TO MIXED MOTIVES

| | | |
|-----------|---------------------------------------|------------|
| 19 | Operations of Correspondences | 172 |
| 19.1 | Cycle Class | 172 |
| 19.2 | Definition of the Operation | 174 |

| | |
|--|------------|
| 20 Grothendieck Motives | 177 |
| 20.1 The Subcategory of Splittable Objects | 178 |
| 20.2 Pure Motives | 180 |
| 21 Polarizability | 182 |
| 21.1 P-Structures | 182 |
| 21.2 The Pure Parts of the Realization of a Variety | 184 |
| 22 Mixed Motives | 186 |
| 22.1 An Approximation of the Category of Mixed Motives | 186 |
| 22.2 The Category $D_{\widetilde{MM}}$ | 189 |
| 22.3 Some Properties | 192 |
| | |
| References | 197 |
| List of Notations | 202 |
| Index | 204 |