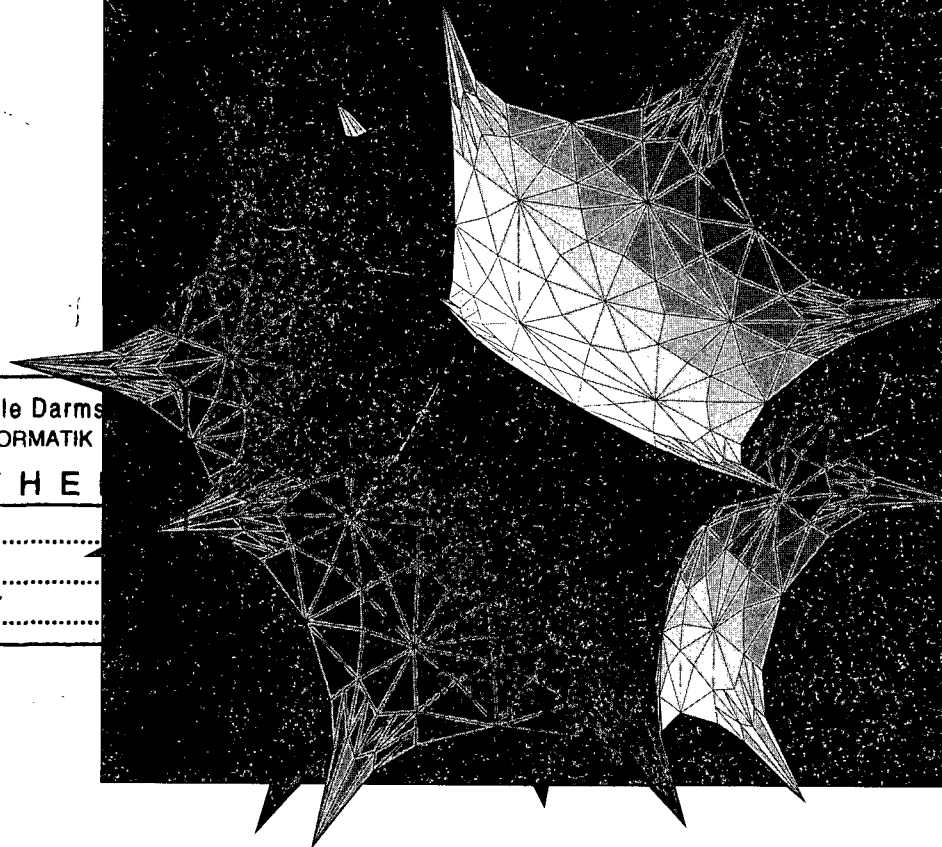


Stephen Wolfram

# Mathematica<sup>®</sup>

A System for Doing Mathematics by Computer

Second Edition



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