

UNCERTAINTY MODELS FOR KNOWLEDGE-BASED SYSTEMS

A Unified Approach to the Measurement of Uncertainty

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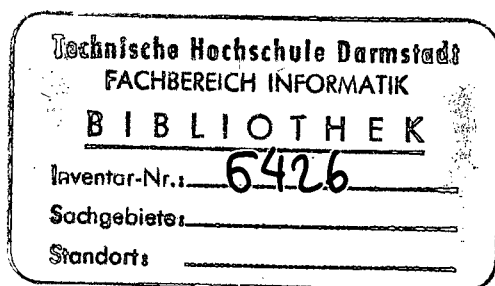


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