

MATHEMATICS FOR PHYSICS

A Guided Tour for Graduate Students

MICHAEL STONE

University of Illinois at Urbana-Champaign

and

PAUL GOLDBART

University of Illinois at Urbana-Champaign



CAMBRIDGE
UNIVERSITY PRESS

Contents

Preface	<i>page xi</i>
Acknowledgments	xiii
1 Calculus of variations	1
1.1 What is it good for?	1
1.2 Functionals	1
1.3 Lagrangian mechanics	10
1.4 Variable endpoints	27
1.5 Lagrange multipliers	32
1.6 Maximum or minimum?	36
1.7 Further exercises and problems	38
2 Function spaces	50
2.1 Motivation	50
2.2 Norms and inner products	51
2.3 Linear operators and distributions	66
2.4 Further exercises and problems	76
3 Linear ordinary differential equations	86
3.1 Existence and uniqueness of solutions	86
3.2 Normal form	93
3.3 Inhomogeneous equations	94
3.4 Singular points	97
3.5 Further exercises and problems	98
4 Linear differential operators	101
4.1 Formal vs. concrete operators	101
4.2 The adjoint operator	104
4.3 Completeness of eigenfunctions	117
4.4 Further exercises and problems	132
5 Green functions	140
5.1 Inhomogeneous linear equations	140
5.2 Constructing Green functions	141

5.3 Applications of Lagrange's identity	150
5.4 Eigenfunction expansions	153
5.5 Analytic properties of Green functions	155
5.6 Locality and the Gelfand–Dikii equation	165
5.7 Further exercises and problems	167
6 Partial differential equations	174
6.1 Classification of PDEs	174
6.2 Cauchy data	176
6.3 Wave equation	181
6.4 Heat equation	196
6.5 Potential theory	201
6.6 Further exercises and problems	224
7 The mathematics of real waves	231
7.1 Dispersive waves	231
7.2 Making waves	242
7.3 Nonlinear waves	246
7.4 Solitons	255
7.5 Further exercises and problems	260
8 Special functions	264
8.1 Curvilinear coordinates	264
8.2 Spherical harmonics	270
8.3 Bessel functions	278
8.4 Singular endpoints	298
8.5 Further exercises and problems	305
9 Integral equations	311
9.1 Illustrations	311
9.2 Classification of integral equations	312
9.3 Integral transforms	313
9.4 Separable kernels	321
9.5 Singular integral equations	323
9.6 Wiener–Hopf equations I	327
9.7 Some functional analysis	332
9.8 Series solutions	338
9.9 Further exercises and problems	342
10 Vectors and tensors	347
10.1 Covariant and contravariant vectors	347
10.2 Tensors	350
10.3 Cartesian tensors	362
10.4 Further exercises and problems	372

11 Differential calculus on manifolds	376
11.1 Vector and covector fields	376
11.2 Differentiating tensors	381
11.3 Exterior calculus	389
11.4 Physical applications	395
11.5 Covariant derivatives	403
11.6 Further exercises and problems	409
12 Integration on manifolds	414
12.1 Basic notions	414
12.2 Integrating p -forms	417
12.3 Stokes' theorem	422
12.4 Applications	424
12.5 Further exercises and problems	440
13 An introduction to differential topology	449
13.1 Homeomorphism and diffeomorphism	449
13.2 Cohomology	450
13.3 Homology	455
13.4 De Rham's theorem	469
13.5 Poincaré duality	473
13.6 Characteristic classes	477
13.7 Hodge theory and the Morse index	483
13.8 Further exercises and problems	496
14 Groups and group representations	498
14.1 Basic ideas	498
14.2 Representations	505
14.3 Physics applications	517
14.4 Further exercises and problems	525
15 Lie groups	530
15.1 Matrix groups	530
15.2 Geometry of $SU(2)$	535
15.3 Lie algebras	555
15.4 Further exercises and problems	572
16 The geometry of fibre bundles	576
16.1 Fibre bundles	576
16.2 Physics examples	577
16.3 Working in the total space	591
17 Complex analysis	606
17.1 Cauchy–Riemann equations	606

17.2	Complex integration: Cauchy and Stokes	616
17.3	Applications	624
17.4	Applications of Cauchy's theorem	630
17.5	Meromorphic functions and the winding number	644
17.6	Analytic functions and topology	647
17.7	Further exercises and problems	661
18	Applications of complex variables	666
18.1	Contour integration technology	666
18.2	The Schwarz reflection principle	676
18.3	Partial-fraction and product expansions	687
18.4	Wiener–Hopf equations II	692
18.5	Further exercises and problems	701
19	Special functions and complex variables	706
19.1	The Gamma function	706
19.2	Linear differential equations	711
19.3	Solving ODEs via contour integrals	718
19.4	Asymptotic expansions	725
19.5	Elliptic functions	735
19.6	Further exercises and problems	741
A	Linear algebra review	744
A.1	Vector space	744
A.2	Linear maps	746
A.3	Inner-product spaces	749
A.4	Sums and differences of vector spaces	754
A.5	Inhomogeneous linear equations	757
A.6	Determinants	759
A.7	Diagonalization and canonical forms	766
B	Fourier series and integrals	779
B.1	Fourier series	779
B.2	Fourier integral transforms	783
B.3	Convolution	786
B.4	The Poisson summation formula	792
	References	797
	Index	799