

Geometrodynamics of Gauge Fields

On the Geometry of Yang-Mills
and Gravitational Gauge Theories

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With 11 Figures



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