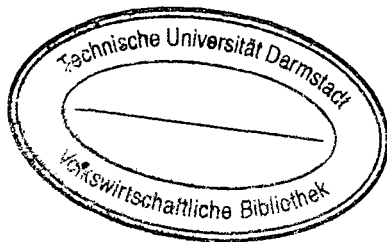


# Time Series Analysis

## Nonstationary and Noninvertible Distribution Theory

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