NOTES AND PROBLEMS IN MICROECONOMIC THEORY

PETER B. DIXON
La Trobe University

SAMUEL BOWLES
University of Massachusetts at Amherst

DAVID KENDRICK
University of Texas at Austin

in collaboration with

LANCE TAYLOR
Massachusetts Institute of Technology

MARC ROBERTS
Harvard University

NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM · NEW YORK · OXFORD

CONTENTS

Introduction XI

Introduction to the Markham edition XV

Chapter 1. Notes and problems in the theory of mathematical programming 1

1.1. Introduction 1
1.2. Goals, reading guide and references 2
1.3. Formal statement of the problem 4
1.4 The gradient vector and contour diagrams 6
1.5. The necessary conditions for a constrained maximum 8
1.6 The role of convexity 15
1.7. The Lagrangian function 18
1.8. Displacement analysis and the interpretation of the Lagrangian multipliers 21

Problem set 1 24

Exercise 1.1. Standard form 24
Exercise 1.2. Contour-gradient diagram 24
Exercise 1.3. Necessary conditions for a constrained maximum 25
Exercise 1.4. An easy optimization problem 28
Exercise 1.5. A more difficult optimization problem 29
Exercise 1.6. An exceptional case 34
Exercise 1.7. Direction of steepest ascent 37
Exercise 1.8. Sign constraints 38
Exercise 1.9. Equality constraints 40
Exercise 1.10. Linear constraints 43
Exercise 1.11. Linear objective function 44
Exercise 1.12. A nonquasiconcave objective function 44
Exercise 1.13. Nonuniqueness of the problem solutions 45
Exercise 1.14. Linear programming, primal and dual 47
Exercise 1.15. On the interpretation of the Lagrangian multipliers 51
Exercise 1.16. An example of nonunique Lagrangian multipliers 52
Exercise 1.17. A problem with multiple unconnected solutions 55
Exercise 1.18. The interpretation of the Lagrangian multipliers when there are inequality constraints 59
Exercise 1.19. Restrictions on the domain of f and g 62

Chapter 2. Theory of the consumer: introduction 65

2.1. Goals, reading guide and references 65
2.2. Notes on utility maximizing 69
2.3. Systems of demand equations 73
2.4 The implications of utility maximizing for demand systems 75

Problem set 2 77

Exercise 2.1. The Engel aggregation 77
Exercise 2.2. Homogeneity restriction 78
Exercise 2.3. The linear expenditure system 79
Exercise 2.4. The marginal utility of income 84
Exercise 2.5. Displacement analysis, an example 86
Exercise 2.6. Displacement analysis and the symmetry restriction 90
Exercise 2.7. The triad 93
Exercise 2.8. The Cournot aggregation 95
Exercise 2.9. The Hicks–Slutsky partition 96
Exercise 2.10. The negativity of the own-price substitution effect 97
Exercise 2.11. The inferiority of Giffen goods 101
Exercise 2.12. The sign of cross-elasticities of demand 102
Exercise 2.13. Substitutes, the two-good case 103
Exercise 2.14. Consumer behavior under rationing 104
Exercise 2.15. The allocation of time 107
Exercise 2.16. Pure exchange and Pareto optimality 109
Exercise 2.17. Additive utility functions 114

Chapter 3. Theory of the consumer: extensions 123

3.1. Goals, reading guide and references 123

Problem set 3 131

A. Problems on revealed preference 131

Exercise 3.1. Revealed preference for an individual 131
Exercise 3.2. Community welfare decisions (a revealed preference problem) 133
Exercise 3.3. International welfare comparisons 135
Exercise 3.4. Revealed preference versus utility maximization 137

B. Problems using the concept of economic surplus 141

Exercise 3.5. A consumer surplus approach to the theater problem 141
Exercise 3.6. The application of the consumer surplus concept to social decisions through cost–benefit analysis (For discussion) 143
Exercise 3.7. Compensated demand functions, consumer surplus and excess burden 144
Exercise 3.8. The costs of protection 147
Exercise 3.9. The costs of average cost pricing 153

Exercise 3.10. Economies of Scale, intraindustry specialization and the costs of protection 157

C. Some difficulties concerning the use of micro restrictions at the macro level 165

Exercise 3.11. Aggregation across households 165
Exercise 3.12. Homogeneity and aggregate demand functions 168
Exercise 3.13. The additivity assumption and aggregation across households 169

D. An integrability proposition 173

Exercise 3.14. The triad and integrability 173

E. Obstacles to Pareto optimal market solutions 181

Exercise 3.15. Interdependent utilities 181
Exercise 3.16. Alliance military expenditures as a public good 183

F. Intertemporal consumer behavior 185

Exercise 3.17. The elimination of stock variables from demand equations for durables 186
Exercise 3.18. Savings as a function of the rate of interest 188
Exercise 3.19. Intertemporal consumption and investment 190
Exercise 3.20. Vicious circles 193
Exercise 3.21. Terminal conditions for multiperiod consumption models 195
Exercise 3.22. Intertemporal inconsistency 202

G. Decisions under uncertainty 207

Exercise 3.23. The determination of utility functions for risky decisions 207
Exercise 3.24. The St. Petersberg paradox 212
Exercise 3.25. The boundedness of the utility function 213
Exercise 3.26. Measurable utility (a review question) 214

Chapter 4. Production theory 215

4.1. Goals, reading guide and references 215
4.2. Background notes on some recent developments in production function theory 221

Problem set 4 224

A. Some properties of production functions 224

Exercise 4.1. Properties of two-factor, linearly homogeneous production functions 224
Exercise 4.2. Factor price frontier 230
Exercise 4.3. Properties of the Cobb–Douglas production function 231
Exercise 4.4. A 'Generalized' Cobb–Douglas production function 234

B. Static optimization 235

Exercise 4.5. Optimization subject to neoclassical technology 235
Exercise 4.6. Optimization in a two-sector-model 239
Exercise 4.7. A planning problem 240
Exercise 4.8. The construction of the production possibilities frontier from a set of production functions and resource endowments 242
Exercise 4.9. The construction of a production function 247
Exercise 4.10. Class interests and induced technical change 250
Exercise 4.11. Technological change, factor supplies and income distribution 254

C. An introduction to linear models 258

Exercise 4.12. Optimization with linear technologies 258
Exercise 4.13. An illustration of Samuelson's (non)substitution theorem 264
Exercise 4.14. Income distribution in Ricardia 267
Exercise 4.15. A factor-price frontier for an economy with linear technologies 269

D. Theoretical developments underlying modern production function econometrics 273

Exercise 4.16. Some formal analysis of the standard cost minimizing model 273
Exercise 4.17. Some properties of cost functions 279
Exercise 4.18. The Allen definition of the elasticity of substitution 288
Exercise 4.19. Alternative definitions of the elasticity of substitution 291
Exercise 4.20. The CES production function 295
Exercise 4.21. The CRESH production function 301
Exercise 4.22. An example of a flexible functional form: the generalized Leontief function 307
Exercise 4.23. Direct specifications of the input demand functions 311

Index 313