

# Lecture Notes in Economics and Mathematical Systems

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## Semi-Infinite Programming and Applications

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