Principles of Random Signal Analysis and Low Noise Design

The Power Spectral Density and its Applications

Roy M. Howard
Curtin University of Technology
Perth, Australia

WILEY-INTERSCIENCE
A JOHN WILEY & SONS, INC., PUBLICATION

Contents

Preface ix

About the Author xi

1. Introduction 1

2. Background: Signal and System Theory 3

2.1 Introduction / 3
2.2 Background Theory / 3
2.3 Functions, Signals and Systems / 7
2.4 Signal Properties / 12
2.5 Measure and Lebesgue Integration / 23
2.6 Signal Classification / 35
2.7 Convergence / 36
2.8 Fourier Theory / 38
2.9 Random Processes / 44
2.10 Miscellaneous Results / 45
Appendix 1: Proof of Theorem 2.11 / 46
Appendix 2: Proof of Theorem 2.13 / 47
Appendix 3: Proof of Theorem 2.17 / 47
Appendix 4: Proof of Theorem 2.27 / 49
Appendix 5: Proof of Theorem 2.28 / 50
Appendix 6: Proof of Theorem 2.30 / 52
Appendix 7: Proof of Theorem 2.31 / 53
Appendix 8: Proof of Theorem 2.32 / 56

3. The Power Spectral Density 59

3.1 Introduction / 59
3.2 Definition / 60
3.3 Properties / 65
3.4 Random Processes / 67
3.5 Existence Criteria / 73
3.6 Impulsive Case / 74
3.7 Power Spectral Density via Autocorrelation / 78
Appendix 1: Proof of Theorem 3.4 / 84
Appendix 2: Proof of Theorem 3.5 / 85
Appendix 3: Proof of Theorem 3.8 / 88
Appendix 4: Proof of Theorem 3.10 / 89

4. Power Spectral Density Analysis 92

4.1 Introduction / 92
4.2 Boundedness of Power Spectral Density / 92
4.3 Power Spectral Density via Signal Decomposition / 95
4.4 Simplifying Evaluation of Power Spectral Density / 98
4.5 The Cross Power Spectral Density / 102
4.6 Power Spectral Density of a Sum of Random Processes / 107
4.7 Power Spectral Density of a Periodic Signal / 112
4.8 Power Spectral Density—Periodic Component Case / 119
4.9 Graphing Impulsive Power Spectral Densities / 122
Appendix 1: Proof of Theorem 4.2 / 123
Appendix 2: Proof of Theorem 4.4 / 126
Appendix 3: Proof of Theorem 4.5 / 128
Appendix 4: Proof of Theorem 4.6 / 128
Appendix 5: Proof of Theorem 4.8 / 130
Appendix 6: Proof of Theorem 4.10 / 132
Appendix 7: Proof of Theorem 4.11 / 134
Appendix 8: Proof of Theorem 4.12 / 136

5. Power Spectral Density of Standard Random Processes—Part 1 138

5.1 Introduction / 138
5.2 Signaling Random Processes / 138
5.3 Digital to Analogue Converter Quantization / 152
5.4 Jitter / 155
5.5 Shot Noise / 160
5.6 Generalized Signaling Processes / 166
Appendix 1: Proof of Theorem 5.1 / 168
Appendix 2: Proof of Theorem 5.2 / 171
Appendix 3: Proof of Equation 5.73 / 173
Appendix 4: Proof of Theorem 5.3 / 174

Appendix 5: Proof of Theorem 5.4 / 176
Appendix 6: Proof of Theorem 5.5 / 177

6. Power Spectral Density of Standard Random Processes—Part 2 **179**

6.1 Introduction / 179
6.2 Sampled Signals / 179
6.3 Quadrature Amplitude Modulation / 185
6.4 Random Walks / 192
6.5 1/f Noise / 198
Appendix 1: Proof of Theorem 6.1 / 200
Appendix 2: Proof of Theorem 6.2 / 201
Appendix 3: Proof of Theorem 6.3 / 202
Appendix 4: Proof of Equation 6.39 / 204

7. Memoryless Transformations of Random Processes **206**

7.1 Introduction / 206
7.2 Power Spectral Density after a Memoryless Transformation / 206
7.3 Examples / 211
Appendix 1: Proof of Theorem 7.1 / 223
Appendix 2: Fourier Results for Raised Cosine Frequency Modulation / 224

8. Linear System Theory **229**

8.1 Introduction / 229
8.2 Impulse Response / 230
8.3 Input-Output Relationship / 232
8.4 Fourier and Laplace Transform of Output / 232
8.5 Input-Output Power Spectral Density Relationship / 238
8.6 Multiple Input-Multiple Output Systems / 243
Appendix 1: Proof of Theorem 8.1 / 246
Appendix 2: Proof of Theorem 8.2 / 248
Appendix 3: Proof of Theorem 8.3 / 249
Appendix 4: Proof of Theorem 8.4 / 251
Appendix 5: Proof of Theorem 8.6 / 252
Appendix 6: Proof of Theorem 8.7 / 253
Appendix 7: Proof of Theorem 8.8 / 255

9. Principles of Low Noise Electronic Design **256**

9.1 Introduction / 256
9.2 Gaussian White Noise / 259
9.3 Standard Noise Sources / 264
9.4 Noise Models for Standard Electronic Devices / 266
9.5 Noise Analysis for Linear Time Invariant Systems / 269

9.6 Input Equivalent Current and Voltage Sources / 278
9.7 Transferring Noise Sources / 282
9.8 Results for Low Noise Design / 285
9.9 Noise Equivalent Bandwidth / 285
9.10 Power Spectral Density of a Passive Network / 287
Appendix 1: Proof of Theorem 9.2 / 291
Appendix 2: Proof of Theorem 9.4 / 294
Appendix 3: Proof of Conjecture for Ladder Structure / 296

Notation **300**

References **302**

Index **307**