Representations of *-Algebras, Locally Compact Groups, and Banach *-Algebraic Bundles

Volume 2
Banach *-Algebraic Bundles, Induced Representations, and the Generalized Mackey Analysis

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