

# Exercises in Econometrics

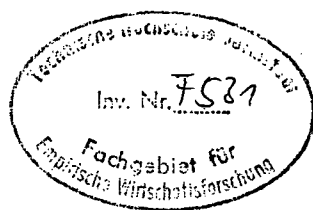
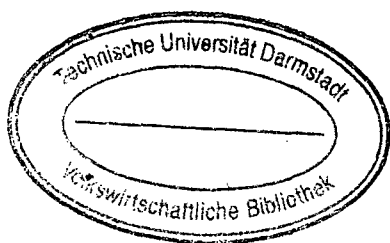
**P. C. B. PHILLIPS**

*Professor of Econometrics, University of Birmingham*

**M. R. WICKENS**

*Reader in Econometrics, University of Essex*

## Volume I



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