

PETER MARTI

THEORY OF STRUCTURES

**FUNDAMENTALS
FRAMED STRUCTURES
PLATES AND SHELLS**

WILEY



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