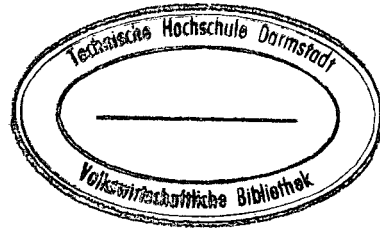


COMPETITIVE EQUILIBRIUM

Theory and Applications

BRYAN ELLICKSON

University of California, Los Angeles



CAMBRIDGE
UNIVERSITY PRESS

Contents

<i>List of Illustrations</i>	page xi
<i>Preface</i>	xv
<i>Acknowledgments</i>	xix
1 Exchange	1
1.1 Mathematical prerequisites	2
1.1.1 Sets	2
1.1.2 Functions	4
1.1.3 Vector spaces	5
1.1.4 Linear functionals, hyperplanes, and halfspaces	9
1.2 Walrasian equilibrium	12
1.2.1 The geometry of exchange	16
1.2.2 Generalizing the net trade diagram	21
1.2.3 Walrasian equilibrium and net trades	23
1.3 Pareto optimality and the core	24
1.3.1 Pareto optimality	25
1.3.2 The core	30
1.4 A numerical example	34
1.4.1 Walrasian equilibrium, Pareto optima, and the core	34
1.4.2 Core equivalence	37
1.5 Commodities and prices	41
1.5.1 Contingent commodities	42
1.5.2 Overlapping generations	44
1.6 Summary	48
2 Production	55
2.1 Geometry of vector spaces	56
2.1.1 Convexity	56

2.1.2	Linear and affine transformations	60
2.1.3	Cones	63
2.1.4	Recession cones	64
2.1.5	Polar cones	66
2.2	CRS production	67
2.2.1	Activity vectors	68
2.2.2	Technology sets	68
2.2.3	Sup and inf	69
2.2.4	Prices and profit	70
2.2.5	Consumption sets	73
2.2.6	Walrasian equilibrium	74
2.2.7	The net trade diagram	75
2.2.8	Pareto optimality	77
2.2.9	The core	78
2.3	Alternative models of production	80
2.3.1	The Arrow-Debreu economy	80
2.3.2	Coalition production economies	83
2.4	Public goods and joint supply	84
2.4.1	Marshallian joint supply	84
2.4.2	Public goods	87
2.4.3	What is competition?	88
2.5	Summary	94
3	Aumann's model	99
3.1	Applying Aumann's model	100
3.1.1	A simple illustration	100
3.1.2	A type economy	104
3.2	Dealing with nonconvexity	106
3.2.1	Nonconvex preferences	107
3.2.2	A market for automobiles	108
3.2.3	A market for houses	111
3.3	Measure and integration	114
3.3.1	The Riemann integral	114
3.3.2	The Lebesgue integral	115
3.3.3	Lebesgue measure	120
3.3.4	Abstract measure and integration	121
3.3.5	Nonatomic measure spaces	123
3.3.6	Formalizing Aumann's model	124
3.3.7	Economies in distribution form	127
3.4	Hedonic theory and local public goods	132

3.4.1	Commodity bundles as measures	132
3.4.2	Hedonic theory	135
3.4.3	Tiebout equilibrium	138
3.5	Summary	145
4	Topology	151
4.1	Introduction to topology	152
4.1.1	Topological spaces	152
4.1.2	The Euclidean topology	154
4.1.3	Subspace and product topologies	157
4.1.4	Metric topologies	159
4.1.5	Convergence	162
4.1.6	Interior, closure, and boundary	165
4.1.7	Continuity	167
4.1.8	Homeomorphisms	169
4.1.9	Connectedness and compactness	170
4.2	Topologies on vector spaces	174
4.2.1	Topological vector spaces	174
4.2.2	Topological taxonomy for TVS's	177
4.2.3	Topological dual spaces	180
4.2.4	Separation and support by hyperplanes	182
4.3	Summary	185
5	Best response	191
5.1	Preferences	192
5.1.1	Binary relations and orderings	192
5.1.2	Neoclassical preferences	195
5.2	Existence of best response	201
5.3	Continuity of best response	207
5.3.1	Graph of a preference relation	207
5.3.2	Topologies on spaces of subsets	211
5.3.3	Continuity of correspondences	214
5.3.4	Uhc and closed graph	217
5.3.5	The Maximum Theorem	218
5.3.6	Applying the Maximum Theorem	222
5.4	Miscellany	226
5.5	Summary	228
6	Clearing markets	233
6.1	Homogeneity	234

6.2	Existence of Walrasian equilibrium	237
6.2.1	The Brouwer Fixed Point Theorem	238
6.2.2	Free disposal	240
6.2.3	Simplices	243
6.2.4	The KKM Theorem	248
6.3	Computation of equilibria	252
6.3.1	Simplicial subdivisions and Sperner's Lemma	252
6.3.2	An algorithm	256
6.3.3	Choosing labels	261
6.3.4	Proof of KKM and Brouwer	264
6.3.5	The Merrill restart algorithm	265
6.3.6	Some myths about Scarf's algorithm	271
6.4	The Excess Demand Theorem	273
6.5	Kakutani Fixed Point Theorem	274
6.6	Summary	277
7	Walras meets Nash	283
7.1	Noncooperative game theory	283
7.1.1	Nash equilibrium	283
7.1.2	Cournot oligopoly	285
7.1.3	Abstract economies	287
7.1.4	Existence of equilibrium	288
7.2	Walrasian equilibrium	290
7.2.1	The Arrow-Debreu model	290
7.2.2	Elaborations	296
7.3	External effects	297
7.3.1	Existence	298
7.3.2	Bees and blossoms	300
7.3.3	Tragedy of the commons	302
7.3.4	A matter of optimality	303
7.4	Nonconvexity	306
7.4.1	The Shapley-Folkman Theorem	306
7.4.2	Nonconvexity and equilibrium	310
7.4.3	The meaning of approximation	314
7.5	Nonordered preferences	314
7.5.1	Existence	315
7.5.2	Drugs and intransitivity	316
7.5.3	Unmasking a cusp catastrophe	322
7.6	Summary	326

8	What is competition?	331
8.1	The Second Fundamental Theorem	331
	8.1.1 Pure exchange	332
	8.1.2 Production	338
8.2	Core equivalence	340
	8.2.1 The Debreu-Scarf Theorem	341
	8.2.2 Core convergence	345
	8.2.3 The integral of a correspondence	350
	8.2.4 Aumann's Theorem	352
8.3	Infinite dimensional commodity spaces	353
	8.3.1 Vector spaces	354
	8.3.2 Linear functionals	356
	8.3.3 Dual pairs and dual topologies	358
	8.3.4 The economic significance of topology	360
	8.3.5 Walrasian equilibrium	363
	8.3.6 Proving existence	364
	8.3.7 Nonempty interior	366
	8.3.8 Properness	367
	8.3.9 A lattice structure for prices	370
8.4	The large square economy	370
8.5	Summary	371
	<i>Bibliography</i>	375
	<i>Index</i>	387