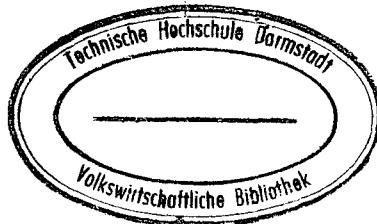


# **ECONOMETRIC ANALYSIS**

**Second Edition**

**William H. Greene**

New York University



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