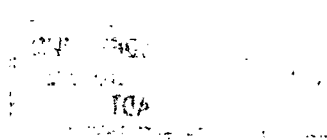


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# 2D-Gravity in Non-Critical Strings

Discrete and Continuum Approaches



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