

# Optimal Control and System Theory in Dynamic Economic Analysis

Masanao Aoki

University of California, Los Angeles



North-Holland  
PUBLISHING COMPANY  
New York • Oxford • Amsterdam



# Contents

<b>Editors Preface</b>	xi
<b>Preface</b>	xiii
<b>PART I STATE SPACE</b>	
<b>1. Dynamic Models</b>	
1.1 Introduction	3
1.2 Dynamic Models in Economics	7
1.3 Dynamic Models and Their Representation	10
<b>2. State Space Models</b>	
2.1 Second-Order Equation	13
2.2 $n$ -th Order Equation	20
2.3 Transform of the State Space Equation	42
2.4 Target-Instrument Pair Representation	51
2.5 Bilinear Dynamic Systems	58
2.6 Linearized Approximations of Nonlinear Dynamic Systems about Reference Path	59
<b>3. Properties of Dynamic Systems:     Controllability and Observability</b>	
3.1 Uncontrollable Dynamic Systems—An Example	72
3.2 Controllability of Linear Dynamic Economic Systems	78
3.3 Classification of Dynamic Systems by Tracking Capability	97
3.4 Local Controllability of Non- linear Discrete Time Dynamic Systems	101
3.5 Controllability of Bilinear Systems	104
3.6 Observability	108
<b>4. Stability of Dynamic Systems</b>	
4.1 Continuous Time Systems	119
4.2 Discrete Time Systems	123
4.3 Time-Varying Systems	127
4.4 Discussion	128



## **PART II     DETERMINISTIC DYNAMIC ECONOMIC SYSTEMS**

<b>5.</b>	<b>Stabilization and Control of Economic Systems</b>	
5.1	Stabilizability	134
5.2	Instrument Instability	137
5.3	Quadratic Welfare Function	156
5.4	Linear Continuous Time Dynamic Systems and the Riccati Equation	160
5.5	Examples	169
<b>6.</b>	<b>Stabilization of a Decentralized System: Example-Open Economy Model</b>	
6.1	Introduction	175
6.2	Model	177
6.3	Stabilization of a Two-country Model	179
6.4	Assignment of Instruments or Decoupling Problem in Decentralized Dynamic Systems	187
<b>7.</b>	<b>Stability Analysis of Dynamic Adjustment Models</b>	
7.1	A Marshallian Adjustment Scheme: Case of Perishable Good	193
7.2	A Monetary Disequilibrium Model: An Example of Stability Analysis of a Nonlinear Economic System	202
	Appendix 7-A: Rationing	229
	Appendix 7-B: Dynamic Equation for Nominal Excess Demand	231
	Appendix 7-C: Computation of Eigenvalues of $F$	232
	Appendix 7-D: Nonzero Equilibrium Excess Demands Under Price Control	232
<b>8.</b>	<b>Policy Mixes, Noninteracting Control and the Assignment Problem</b>	
8.1	Introductory Examples	235
8.2	Criteria of Noninteraction	243
8.3	Imperfectly Known Dynamic System	251
8.4	Example	255



## PART III      STOCHASTIC DYNAMIC ECONOMIC SYSTEMS

<b>9.</b>	<b>Autoregressive Moving Average Processes</b>	
9.1	Introduction	267
9.2	Orthogonal Projections	267
9.3	State Space Representation	269
9.4	Controllability in ARMA Process	271
9.5	Examples	272
<b>10.</b>	<b>Regulation and Models of Stochastic Dynamic Economies</b>	
10.1	Kalman Filter	278
10.2	Linear Regulator Problems with Quadratic Cost Function	280
10.3	Aggregation	286
	Appendix: Aggregation	294
<b>11.</b>	<b>Random Multiplier—An Example of Control of Stochastic Dynamic Macroeconomic Model</b>	
11.1	Introduction	297
11.2	Random Money Multiplier	298
11.3	Control of a Linear Macroeconomic Model	302
11.4	Discussion	306
<b>12.</b>	<b>Price and Quantity Adjustments Schemes</b>	
12.1	Price Adjustment and Cobweb Theorem	307
12.2	A Stochastic Marshallian Quantity Adjustment	319
12.3	Pricing Policy of a Marketeer	325
	Appendix A:	332
	Appendix B: Convergence in Probability	337
	Appendix C: Convergence in Mean Square	339
	Appendix D: Convergence with Probability One	342
<b>13.</b>	<b>Price Adjustment of a Middleman with Inventory</b>	
13.1	Introduction	343
13.2	Model	344
13.3	Analysis	349
	Appendix A: Computations of $E(T_Q)$ and $\text{var}(T_Q)$	359



Appendix B:	Derivation of the Beta Distributions	363
Appendix C:	Computation of $E(S) = EV_1/ET_1$	364
Appendix D:	Optimal Price for a Given $Q$	366
Appendix E:	Maximum Likelihood Estimates and Asymptotic Distributions	367
<b>Epilogue</b>		371
<b>References</b>		373
<b>Appendix A:</b>	<b>Functions of Matrices</b>	385
<b>Appendix B:</b>	<b>Some Useful Relations</b>	389
<b>Appendix C:</b>	<b>Approximate Determination of Roots of a Polynomial</b>	391
<b>Appendix D:</b>	<b>Root-Locus Method of Stability Analysis</b>	393
<b>Index</b>		397