

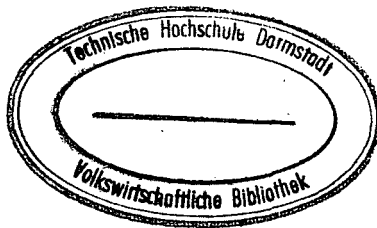
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Second Edition

# The Asymptotic Theory of Extreme Order Statistics

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