Hydraulic Fracture Mechanics

1

Peter Valkó and Michael J. Economides

Texas A & M University, College Station, USA

JOHN WILEY & SONS Chichester • New York • Brisbane • Toronto • Singapore

CONTENTS

Preface			xi	
List	List of Notation			
1	Hydr	aulically Induced Fractures in the Petroleum		
		Related Industries	1	
			-	
		Fractures in Well Stimulation	1	
	1.2	Fluid Flow Through Porous Media	2	
	1.3	1.2.1 The Near-well Zone Flow from a Fractured Well	4 5 7	
		Hydraulic Fracture Design	5	
	1.5	, ,	11	
		1.5.1 Fracturing Fluids	11	
		1.5.2 Proppants	13	
	1.6	Data Acquisition and Evaluation for Hydraulic Fracturing	14	
		1.6.1 Well Log Measurements	14	
		1.6.2 Core Measurements	15 15	
	1.7	1.6.3 Well Testing Mechanics in Hydraulic Fracturing	15	
		References	16	
•				
2		ar Elasticity, Fracture Shapes and		
	Indu	ced Stresses	19	
	2.1	Force and Deformation	19	
		2.1.1 Stress	19	
		2.1.2 Strain	21	
	2.2		23	
		2.2.1 Linear Elastic Material	23 26	
	2.3	2.2.2 Material Behavior Beyond Perfect Elasticity Plane Elasticity	26 27	
	2.0	2.3.1 Plane Stress	27	
		2.3.2 Stresses Relative to an Oblique Line		
		(Force Balance I)	28	
		2.3.3 Equilibrium Relations (Force Balance II)	30	
		2.3.4 Plane Strain	30	
		2.3.5 Boundary Conditions	32	

	2.4	Pressurized Crack 2.4.1 Solution of the Line Crack Problem 2.4.2 Constant Pressure 2.4.3 Polynomial Pressure Distribution 2.4.4 "Zipper" Cracks 2.4.5 "Zipper" Crack with Polynomial Pressure Distribution	32 32 34 35 37 40
	2.5 2.6	 Stress Concentration and Stress Intensity Factor 2.5.1 Stress Intensity Factor, Symmetric Loading 2.5.2 Stress Intensity Factor, non-symmetric Loading Fracture Shape in the Presence of Far-field Stress. 	41 42 43
	2.0	The Concept of Net Pressure	43
	2.7	Circular Crack	45
	2.8	Volume and Strain Energy	47
	2.9	Computational Methods References	49 50
3	Stres	sses in Formations	53
	3.1	Basic Concepts	53
		Stresses at Depth	55
	3.3	Near-wellbore Stresses	59
	3.4	Stress Concentrations for an Arbitrarily Oriented Well	63
	3.5	Vertical Well Breakdown Pressure	65
	3.6	Breakdown Pressure for an Arbitrarily Oriented Well	66
	3.7	Limiting Case: Horizontal Well 3.7.1 Arbitrarily Oriented Horizontal Well	69 70
	3.8	Permeability and Stress	71
		3.8.1 Stress-sensitive Permeability	72
	3.9	Measurement of Stresses	73
		3.9.1 Small Interval Fracture Injection Tests	74
		3.9.2 Acoustic Measurements	75
		3.9.3 Determination of the Closure Pressure	76
		3.9.4 Core Stress Measurements	77
		3.9.5 Critique and Applicability of Techniques References	79 80
4	Frac	ture Geometry	83
	4.1	The Perkins and Kern and Khristianovich and	
		Zheltov Geometries	83
		4.1.1 The Consequences of the Plane Strain Assumption	86
	4.2	Fracture Initiation vs. Propagation Direction 4.2.1 Fractures in Horizontal Wells	88 90
	4.3	Fracture Profiles in Multi-layered Formations References	92 95
5	Rhee	blogy and Laminar Flow	97
	5.1		97
	5.1	Basic Concepts 5.1.1 Material Behavior and Constitutive Equations 5.1.2 Force Balance	97 98 103

5.2		105
	5.2.1 Derivation of the Basic Relations	105
	5.2.2 Equivalent Newtonian Viscosity	111
5.3	Flow in Circular Tube	112
	5.3.1 Basic Relations	112
	5.3.2 Flow Curve	115
	5.3.3 Equivalent Newtonian Viscosity for Tube Flow	119
5.4	Flow in Other Cross Sections	122
	5.4.1 Flow in Annulus	122
	5.4.2 Flow in Elliptic Cross Section	123
	5.4.3 Limiting Ellipsoid Cross Section	124
	References	128
Non-	laminar Flow and Solids Transport	131
6.1	Non-laminar Flow	131
	6.1.1 Newtonian Fluid	131
	6.1.2 General Fluid	132
	6.1.3 Drag Reduction	134
	6.1.4 Turbulent Flow in Other Geometries	137

6.2 Solids Transport

Solids 7	138	
6.2.1	Settling of an Individual Sphere	139
6.2.2	Effect of Shear Rate Induced by Flow	141
6.2.3	Effect of Slurry Concentration	142
6.2.4	Wall Effects	143
6.2.5	Agglomeration Effects	145
References		

7 Advanced Topics of Rheology and Fluid Mechanics 147

7.1	Foam F	Rheology	147
	7.1.1	Quality Based Correlations	148
	7.1.2	Volume Equalized Constitutive Equations	148
	7.1.3	Volume Equalized Power Law	151
	7.1.4	Turbulent Flow of Foam	152
7.2	Accoun	ting for Mechanical Energy	153
	7.2.1	Basic Concepts	153
	7.2.2	Incompressible Flow	154
	7.2.3	Foam Flow	154
7.3	Rheometry		156
	7.3.1	Pipe Viscometry	156
	7.3.2	Slip Correction	157
	Referer	162	

8 Material Balance

6

165

8.1	The Co	nservation of Mass and Its Relation to	
	Fracture	e Dimensions	165
8.2	8.2 Fluid Leakoff and Spurt Loss as Material Properties		169
	8.2.1	Carter Equation I	169
	8.2.2	Formal Material Balance. The Opening Time	
		Distribution Factor	171

	8.3 8.4	The Constant Width Approximation (Carter Equation II) The Power Law Approximation to Surface Growth 8.4.1 The Consequences of the Power Law Assumption 8.4.2 The Combination of the Power Law Assumption	172 174 174 178
	05	with Interpolation Numerical Material Balance	179
	8.5		181
	8.6	Differential Material Balance	
	8.7	Leakoff as Flow in the Porous Medium	183
		8.7.1 Filter-cake Pressure Drop	184
		8.7.2 Pressure Drop in the Reservoir	185
		8.7.3 Leakoff Rate from Combining the Resistances (Ehlig-Economides et al. [6])	187
		References	187
9	Cou	pling of Elasticity, Flow and Material Balance	189
	9.1	Width Equations of the Early 2D Models	189
		9.1.1 Perkins~Kern Width Equation	189
		9.1.2 Geertsma-de Klerk Width Equation	192
		9.1.3 Radial Width Equation	195
	9.2	Algebraic (2D) Models as Used in Design	196
		9.2.1 PKN-C	196
		9.2.2 KGD-C	199
		9.2.3 PKN-N and KGD-N	200
		9.2.4 PKN- α and KGD- α	201 202
		9.2.5 Radial Model 9.2.6 Non-Newtonian Behavior	202
	9.3		202
	9.3 9.4		204
	9.4	9.4.1 Nordgren Equation	205
		9.4.2 Differential Horizontal Plane Strain Model	200
	9.5	Models With Detailed Leakoff Description	210
	9.6	Pressure Decline Analysis	211
	5.0	9.6.1 Nolte's Pressure Decline Analysis	211
		(Power Law Assumption)	212
		9.6.2 The No-spurt-loss Assumption	
		(Shlyapobersky method)	217
		9.6.3 Material Balance and Propagation Pressure	
		Estimates of the Spurt Loss	218
		9.6.4 Resolving Contradictions	227
		9.6.5 Pressure Decline Analysis With Detailed Leakoff	
		Description (Mayerhofer et al. Technique)	230
		References	232
10	Frac	ture Propagation	235
	10.1	Fracture Mechanics	237
	10.1	10.1.1 Griffith's Analysis of Crack Stability	238
		10.1.2 Mott's Theory for the Rate of Crack Growth	241
	10.2	•	
		Hydraulic Fracturing	242
		10.2.1 Fracture Toughness Criterion	242
		10.2.2 The Injection Rate Dependence Paradox	243

10.3	Retarde	ed Fracture Propagation	245
	10.3.1	Fluid Lag	245
	10.3.2	Tip Dilatancy	245
	10.3.3	Apparent Fracture Toughness	246
	10.3.4	Process Zone Concept	246
	10.3.5	The Reopening Paradox	247
10.4	Continuum Damage Mechanics in Hydraulic Fracturing		
	10.4.1	Tip Propagation Velocity from CDM	247
	10.4.2	CDM-NK Model	249
	10.4.3	CDM-PKN Design Model	252
10.5	Pressure Decline Analysis and Tip Retardation		
	10.5.1	Resolving Contradictions with Continuum	
		Damage Mechanics	258
	Referer	nces	263

11	Frac	ture Height Growth (3D and P-3D Geometries)	267
	11.1	Equilibrium Fracture Height	269
		11.1.1 Reverse Application of the Net-pressure Concept	269
		11.1.2 Different Systems of Notation	270
		11.1.3 Basic Equations	272
		11.1.4 The Effect of Hydrostatic Pressure	276
	11.2	Three-dimensional Models	278
		11.2.1 Surface Integral Method	279
		11.2.2 The Stress Intensity Factor Paradox	281
	11.3	Pseudo-three-dimensional Models	283
	-	References	284

Appendix: Comparison Study of Hydraulic Fracturing	
Models: Input Data and Results	287
References	294
Index	295