STOCHASTIC MODELS IN OPERATIONS RESEARCH

Volume I

Stochastic Processes and Operating Characteristics

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McGraw-Hill Book Company

New York St. Louis San Francisco Auckland Bogotá Hamburg
Johannesburg London Madrid Mexico Montreal New Delhi
Panama Paris São Paulo Singapore Sydney Tokyo Toronto

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