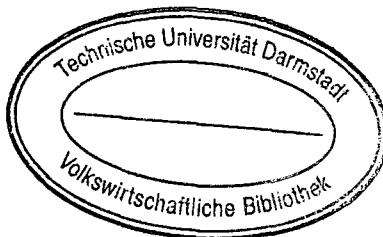


# **Estimation and Inference in Econometrics**

RUSSELL DAVIDSON

JAMES G. MACKINNON



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