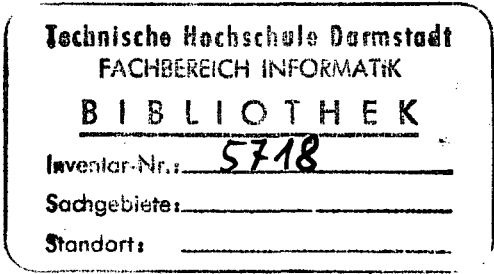


Volume 1 / **Fundamental Algorithms**

**THE ART OF
COMPUTER PROGRAMMING
SECOND EDITION**



Reading, Massachusetts
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