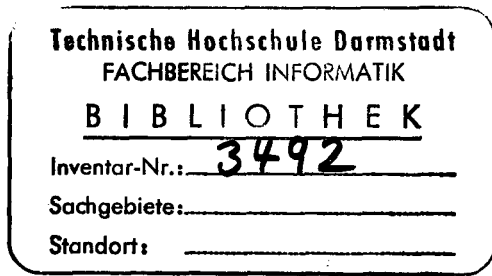


Sequency Theory

Foundations and Applications

HENNING F. HARMUTH

DEPARTMENT OF ELECTRICAL ENGINEERING
THE CATHOLIC UNIVERSITY OF AMERICA
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