

Lecture Notes in Economics and Mathematical Systems

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300

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Programming in Networks and Graphs

On the Combinatorial Background
and Near-Equivalence of
Network Flow and Matching Algorithms



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