

# **Some Modern Mathematics For Physicists and Other Outsiders**

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An Introduction to Algebra, Topology,  
and Functional Analysis  
(Volume 1)

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# Contents of Volume 1

<b>Contents of Volume 2</b>	<b>x</b>
<b>Preface</b>	<b>xiii</b>
<b>Organization of the Book</b>	<b>xvii</b>
<b>Acknowledgments</b>	<b>xxi</b>
<b>Introduction</b>	<b>xxiii</b>

## **PART ONE: THE RAW MATERIALS OF MATHEMATICS 1**

<b>Chapter 1 Sets</b>	<b>3</b>
1.1 Operations with Sets	6
1.2 Relations in Sets	13
1.2a. Equivalence relations	15
1.2b. Order relations	20
<b>Chapter 2 Maps</b>	<b>26</b>
2.1 Composite Functions and Inverses	32
2.2 Equivalence Relations and Maps	38
2.3 Ordered Sets and Maps	42
2.4 Cardinal Numbers	43
2.5 Sequences and Families	47
	<b>vii</b>

<b>PART TWO: THE BASIC STRUCTURES OF MATHEMATICS</b>	<b>53</b>
<i>IIA: Algebraic Structures</i>	
<b>Chapter 3 Algebraic Composition Laws and Systems</b>	<b>57</b>
3.1 Morphisms of Algebraic Systems	62
<b>Chapter 4 Survey of Special Algebraic Systems</b>	<b>68</b>
4.1 Groups	70
4.1a. Transformation groups; $G$ -spaces; orbits	79
4.1b. Conjugate classes; cosets	87
4.1c. Normal subgroups; quotient groups; isomorphism theorems	91
4.2 Rings and Fields	102
4.2a. Ideals; quotient rings; isomorphism theorems	115
4.3 Linear Spaces	120
4.3a. Linear independence, bases and dimension	128
4.3b. Morphisms (linear transformations); quotient spaces	137
4.4 Linear Algebras	149
4.4a. Morphisms of algebras; quotient algebras	159
4.5 Nonassociative Algebras	167
4.5a. Lie algebras	168
4.5b. Some other nonassociative algebras	182
<i>IIB: Topological Structures</i>	
<b>Chapter 5 Topological Spaces</b>	<b>187</b>
5.1 Examples; Metric Spaces	188
5.2 General Structure of Topological Spaces	198
5.3 Neighborhoods; Special Points; Closed Sets	203
5.3a. Interior, closure, boundary	208
5.4 Convergence	211
5.5 Continuity	216
5.6 Homeomorphism and Isometry	220
5.6a. Quotient topology; homeomorphism theorem	229
<b>Chapter 6 Topological Spaces with Special Properties</b>	<b>236</b>
6.1 Connected Spaces	236
6.1a. Path connectivity; homotopy	242
6.2 Separable Spaces	250

6.3	Compact Spaces	254
6.3a.	Compactification	266
6.4	Complete Metric Spaces	270
6.4a.	Completion	275
6.4b.	Contraction mappings	280
	<i>IIC: Measure Structures</i>	291
<b>Chapter 7</b>	<b>Measure Spaces</b>	<b>293</b>
7.1	Measurable Spaces	294
7.2	Measure and Measure Spaces	305
7.2a.	General properties of measures	310
7.2b.	Lebesgue measure	314
7.2c.	Lebesgue–Stieltjes measures	320
7.2d.	Signed and complex measure	325
<b>Chapter 8</b>	<b>Theory of Integration</b>	<b>328</b>
8.1	Measurable Functions	329
8.2	Definition of the Integral	338
8.3	General Properties of the Integral	353
8.4	Comments on Lebesgue and Lebesgue–Stieltjes Integrals	365
8.5	The Radon–Nikodym Theorem	371
	<b>APPENDICES</b>	<b>xxvii</b>
<b>Appendix I</b>	<b>Some Inequalities</b>	<b>xxix</b>
<b>Appendix III</b>	<b>Annotated Reading List</b>	<b>xxx</b>
<b>Appendix IV</b>	<b>Frequently Used Symbols</b>	<b>xxxvi</b>
<b>Index</b>		<b>xliii</b>