Theory of Thin Elastic Shells

M. Dikmen
Technical University of Istanbul

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Technische Hochschule Darmstadt
Fachbereich Mechanik
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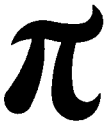

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