Marcel Berger Bernard Gostiaux

Differential Geometry: Manifolds, Curves, and Surfaces

Translated from the French by Silvio Levy

With 249 Illustrations



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Contents

| Preface | v |
|--|-----------|
| Chapter 0. Background | 1 |
| 0.0 Notation and Recap | 2 |
| 0.1 Exterior Algebra | 3 |
| 0.2 Differential Calculus | 9 |
| 0.3 Differential Forms | 17 |
| 0.4 Integration | 25 |
| 0.5 Exercises | 28 |
| Chapter 1. Differential Equations | 30 |
| 1.1 Generalities | 31 |
| 1.2 Equations with Constant Coefficients. Existence of Local | |
| Solutions | 33 |
| 1.3 Global Uniqueness and Global Flows | 38 |
| 1.4 Time- and Parameter-Dependent Vector Fields | 41 |
| 1.5 Time-Dependent Vector Fields: Uniqueness And Global Flow | 43 |
| 1.6 Cultural Digression | 44 |
| Chapter 2. Differentiable Manifolds | 47 |
| 2.1 Submanifolds of \mathbf{R}^n | 48 |
| 2.2 Abstract Manifolds | 54 |
| 2.3 Differentiable Maps | 61 |
| 2.4 Covering Maps and Quotients | 67、 |
| 2.5 Tangent Spaces | 74 |

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| Chapter 3. Partitions of Unity, Densities and Curves1033.1 Embeddings of Compact Manifolds1043.2 Partitions of Unity1063.3 Densities1093.4 Classification of Connected One-Dimensional Manifolds1153.5 Vector Fields and Differential Equations on Manifolds1193.6 Exercises126Chapter 4. Critical Points1284.1 Definitions and Examples1294.2 Non-Degenerate Critical Points1324.3 Sard's Theorem1424.4 Exercises144Chapter 5. Differential Forms1465.1 The Bundle A'T* X1475.2 Differential Forms on a Manifold1485.3 Volume Forms and Orientation1555.4 De Rham Groups1685.5 Lie Derivatives1725.6 Star-shaped Sets and Poincaré's Lemma1765.7 De Rham Groups of Spheres and Projective Spaces1785.8 De Rham Groups of Tori1825.9 Exercises1886.1 Integration of Differential Forms1886.1 Integration of Stokes' Theorem1996.2 Stokes' Theorem1996.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of Tubes II2196.8 Volume of Tubes II2176.9 Volume of Tubes II2276.9 Volume of Tubes II227 <th>2.6 2.7 2.8</th> <th>Submanifolds, Immersions, Submersions and Embeddings Normal Bundles and Tubular Neighborhoods Exercises</th> <th>85 90 96</th> | 2.6 2.7 2.8 | Submanifolds, Immersions, Submersions and Embeddings Normal Bundles and Tubular Neighborhoods Exercises | 85 90 96 |
|---|-------------------|---|----------------|
| 3.1 Embeddings of Compact Manifolds1043.2 Partitions of Unity1063.3 Densities1093.4 Classification of Connected One-Dimensional Manifolds1193.5 Vector Fields and Differential Equations on Manifolds1193.6 Exercises126Chapter 4. Critical Points1284.1 Definitions and Examples1294.2 Non-Degenerate Critical Points1324.3 Sard's Theorem1424.4 Exercises144Chapter 5. Differential Forms1465.1 The Bundle Λ^*T^*X 1475.2 Differential Forms on a Manifold1485.3 Volume Forms and Orientation1555.4 De Rham Groups1685.5 Lie Derivatives1725.6 Star-shaped Sets and Poincaré's Lemma1765.7 De Rham Groups of Spheres and Projective Spaces1785.8 De Rham Groups of Tori1825.9 Exercises184Chapter 6. Integration of Differential Forms1886.1 Integrating Forms of Maximal Degree1956.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^4(X)$ 251 | Chapte | er 3. Partitions of Unity, Densities and Curves | 103 |
| 3.2 Partitions of Unity1063.3 Densities1093.4 Classification of Connected One-Dimensional Manifolds1153.5 Vector Fields and Differential Equations on Manifolds1193.6 Exercises126Chapter 4. Critical Points1284.1 Definitions and Examples1294.2 Non-Degenerate Critical Points1324.3 Sard's Theorem1424.4 Exercises144Chapter 5. Differential Forms1465.1 The Bundle $\Lambda^r T^* X$ 1475.2 Differential Forms on a Manifold1485.3 Volume Forms and Orientation1555.4 De Rham Groups1685.5 Lie Derivatives1725.6 Star-shaped Sets and Poincaré's Lemma1765.7 De Rham Groups of Spheres and Projective Spaces184Chapter 6. Integration of Differential Forms1886.1 Integrating Forms of Maximal Degree1896.2 Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^4(X)$ 251 | 3.1 | Embeddings of Compact Manifolds | 104 |
| 3.3 Densities1093.4 Classification of Connected One-Dimensional Manifolds1153.5 Vector Fields and Differential Equations on Manifolds1193.6 Exercises126Chapter 4. Critical Points1284.1 Definitions and Examples1294.2 Non-Degenerate Critical Points1324.3 Sard's Theorem1424.4 Exercises144Chapter 5. Differential Forms1465.1 The Bundle $\Lambda^r T^* X$ 1475.2 Differential Forms on a Manifold1485.3 Volume Forms and Orientation1555.4 De Rham Groups1685.5 Lie Derivatives1725.6 Star-shaped Sets and Poincaré's Lemma1765.9 Exercises184Chapter 6. Integration of Differential Forms1886.1 Integrating Forms of Maximal Degree1896.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2196.8 Volume of Tubes II2176.9 Volume of Tubes II2136.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^4(X)$ 251 | 3.2 | Partitions of Unity | 106 |
| 3.4 Classification of Connected One-Dimensional Manifolds1153.5 Vector Fields and Differential Equations on Manifolds1193.6 Exercises126Chapter 4. Critical Points1284.1 Definitions and Examples1294.2 Non-Degenerate Critical Points1324.3 Sard's Theorem1424.4 Exercises144Chapter 5. Differential Forms1465.1 The Bundle A^rT^*X 1475.2 Differential Forms on a Manifold1485.3 Volume Forms and Orientation1555.4 De Rham Groups1685.5 Lie Derivatives1725.6 Star-shaped Sets and Poincaré's Lemma1765.9 Exercises184Chapter 6. Integration of Differential Forms1886.1 Integrating Forms of Maximal Degree1896.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of Tubes I2016.6 Canonical Density on a Submanifold of Euclidean Space2176.9 Volume of Tubes II2196.8 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 3.3 | Densities | 109 |
| 3.5 Vector Fields and Differential Equations on Manifolds1193.6 Exercises126Chapter 4. Critical Points1284.1 Definitions and Examples1294.2 Non-Degenerate Critical Points1324.3 Sard's Theorem1424.4 Exercises144Chapter 5. Differential Forms1465.1 The Bundle A^rT^*X 1475.2 Differential Forms on a Manifold1485.3 Volume Forms and Orientation1555.4 De Rham Groups1685.5 Lie Derivatives1725.6 Star-shaped Sets and Poincaré's Lemma1765.7 De Rham Groups of Spheres and Projective Spaces1785.8 De Rham Groups of Tori1825.9 Exercises184Chapter 6. Integration of Differential Forms1886.1 Integrating Forms of Maximal Degree1896.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2196.8 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 3.4 | Classification of Connected One-Dimensional Manifolds | 115 |
| 3.6 Exercises126Chapter 4. Critical Points1284.1 Definitions and Examples1294.2 Non-Degenerate Critical Points1324.3 Sard's Theorem1424.4 Exercises144Chapter 5. Differential Forms1465.1 The Bundle $\Lambda'T^*X$ 1475.2 Differential Forms on a Manifold1485.3 Volume Forms and Orientation1555.4 De Rham Groups1685.5 Lie Derivatives1725.6 Star-shaped Sets and Poincaré's Lemma1765.7 De Rham Groups of Spheres and Projective Spaces1785.8 De Rham Groups of Tori1825.9 Exercises184Chapter 6. Integration of Differential Forms1886.1 Integrating Forms of Maximal Degree1896.2 Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2196.8 Volume of Tubes II2196.9 Volume of Tubes II2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 3.5 | Vector Fields and Differential Equations on Manifolds | 119 |
| Chapter 4. Critical Points1284.1 Definitions and Examples1294.2 Non-Degenerate Critical Points1324.3 Sard's Theorem1424.4 Exercises144Chapter 5. Differential Forms1465.1 The Bundle $\Lambda^r T^* X$ 1475.2 Differential Forms on a Manifold1485.3 Volume Forms and Orientation1555.4 De Rham Groups1685.5 Lie Derivatives1725.6 Star-shaped Sets and Poincaré's Lemma1765.7 De Rham Groups of Spheres and Projective Spaces1785.8 De Rham Groups of Tori1825.9 Exercises184Chapter 6. Integration of Differential Forms1886.1 Integrating Forms of Maximal Degree1896.2 Stokes' Theorem1956.3 First Applications of Stokes' Theorem2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Volume Forms2036.5 Volume of Tubes I2196.8 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 3.6 | Exercises | 126 |
| 4.1 Definitions and Examples1294.2 Non-Degenerate Critical Points1324.3 Sard's Theorem1424.4 Exercises144Chapter 5. Differential Forms1465.1 The Bundle $\Lambda^r T^* X$ 1475.2 Differential Forms on a Manifold1485.3 Volume Forms and Orientation1555.4 De Rham Groups1685.5 Lie Derivatives1725.6 Star-shaped Sets and Poincaré's Lemma1765.7 De Rham Groups of Spheres and Projective Spaces1785.8 De Rham Groups of Tori1825.9 Exercises184Chapter 6. Integration of Differential Forms1886.1 Integrating Forms of Maximal Degree1996.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2196.8 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | Chapte | er 4. Critical Points | 128 |
| 4.2 Non-Degenerate Critical Points1324.3 Sard's Theorem1424.4 Exercises144Chapter 5. Differential Forms1465.1 The Bundle $\Lambda'T^*X$ 1475.2 Differential Forms on a Manifold1485.3 Volume Forms and Orientation1555.4 De Rham Groups1685.5 Lie Derivatives1725.6 Star-shaped Sets and Poincaré's Lemma1765.7 De Rham Groups of Spheres and Projective Spaces1785.8 De Rham Groups of Tori1825.9 Exercises184Chapter 6. Integration of Differential Forms1886.1 Integrating Forms of Maximal Degree1896.2 Stokes' Theorem1956.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2196.8 Volume of Tubes II2196.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 4.1 | Definitions and Examples | 129 |
| 4.3 Sard's Theorem1424.4 Exercises144Chapter 5. Differential Forms1465.1 The Bundle $\Lambda^r T^* X$ 1475.2 Differential Forms on a Manifold1485.3 Volume Forms and Orientation1555.4 De Rham Groups1685.5 Lie Derivatives1725.6 Star-shaped Sets and Poincaré's Lemma1765.7 De Rham Groups of Spheres and Projective Spaces1785.8 De Rham Groups of Tori1825.9 Exercises184Chapter 6. Integration of Differential Forms1886.1 Integrating Forms of Maximal Degree1896.2 Stokes' Theorem1956.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2196.8 Volume of Tubes II2196.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 4.2 | Non-Degenerate Critical Points | 132 |
| 4.4 Exercises144Chapter 5. Differential Forms1465.1 The Bundle $\Lambda^r T^* X$ 1475.2 Differential Forms on a Manifold1485.3 Volume Forms and Orientation1555.4 De Rham Groups1685.5 Lie Derivatives1725.6 Star-shaped Sets and Poincaré's Lemma1765.7 De Rham Groups of Spheres and Projective Spaces1785.8 De Rham Groups of Tori1825.9 Exercises184Chapter 6. Integration of Differential Forms1886.1 Integrating Forms of Maximal Degree1896.2 Stokes' Theorem1956.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of Tubes I2146.7 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 4.3 | Sard's Theorem | 142 |
| Chapter 5. Differential Forms1465.1 The Bundle $\Lambda^r T^* X$ 1475.2 Differential Forms on a Manifold1485.3 Volume Forms and Orientation1555.4 De Rham Groups1685.5 Lie Derivatives1725.6 Star-shaped Sets and Poincaré's Lemma1765.7 De Rham Groups of Spheres and Projective Spaces1785.8 De Rham Groups of Tori1825.9 Exercises184Chapter 6. Integration of Differential Forms1886.1 Integrating Forms of Maximal Degree1896.2 Stokes' Theorem1956.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 4.4 | Exercises | 144 |
| 5.1 The Bundle $\Lambda^r T^* X$ 1475.2 Differential Forms on a Manifold1485.3 Volume Forms and Orientation1555.4 De Rham Groups1685.5 Lie Derivatives1725.6 Star-shaped Sets and Poincaré's Lemma1765.7 De Rham Groups of Spheres and Projective Spaces1785.8 De Rham Groups of Tori1825.9 Exercises184Chapter 6. Integration of Differential Forms6.1 Integrating Forms of Maximal Degree1896.2 Stokes' Theorem1956.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | Chapte | er 5. Differential Forms | 146 |
| 5.2 Differential Forms on a Manifold1485.3 Volume Forms and Orientation1555.4 De Rham Groups1685.5 Lie Derivatives1725.6 Star-shaped Sets and Poincaré's Lemma1765.7 De Rham Groups of Spheres and Projective Spaces1785.8 De Rham Groups of Tori1825.9 Exercises184Chapter 6. Integration of Differential Forms6.1 Integrating Forms of Maximal Degree1896.2 Stokes' Theorem1956.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2196.8 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 5.1 | The Bundle $\Lambda^r T^* X$ | 147 |
| 5.3Volume Forms and Orientation1555.4De Rham Groups1685.5Lie Derivatives1725.6Star-shaped Sets and Poincaré's Lemma1765.7De Rham Groups of Spheres and Projective Spaces1785.8De Rham Groups of Tori1825.9Exercises184Chapter 6. Integration of Differential Forms6.1Integrating Forms of Maximal Degree1896.2Stokes' Theorem1956.3First Applications of Stokes' Theorem1996.4Canonical Volume Forms2036.5Volume of a Submanifold of Euclidean Space2076.6Canonical Density on a Submanifold of Euclidean Space2146.7Volume of Tubes I2196.8Volume of Tubes III2336.10Exercises238Chapter 7. Degree Theory2447.1Preliminary Lemmas2457.2Calculation of $R^d(X)$ 251 | 5.2 | Differential Forms on a Manifold | 148 |
| 5.4 De Rham Groups1685.5 Lie Derivatives1725.6 Star-shaped Sets and Poincaré's Lemma1765.7 De Rham Groups of Spheres and Projective Spaces1785.8 De Rham Groups of Tori1825.9 Exercises184Chapter 6. Integration of Differential Forms6.1 Integrating Forms of Maximal Degree1896.2 Stokes' Theorem1956.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 5.3 | Volume Forms and Orientation | 155 |
| 5.5Lie Derivatives1725.6Star-shaped Sets and Poincaré's Lemma1765.7De Rham Groups of Spheres and Projective Spaces1785.8De Rham Groups of Tori1825.9Exercises184Chapter 6. Integration of Differential Forms6.1Integrating Forms of Maximal Degree1896.2Stokes' Theorem1956.3First Applications of Stokes' Theorem1996.4Canonical Volume Forms2036.5Volume of a Submanifold of Euclidean Space2076.6Canonical Density on a Submanifold of Euclidean Space2146.7Volume of Tubes I2276.9Volume of Tubes III2336.10Exercises238Chapter 7. Degree Theory2447.1Preliminary Lemmas2457.2Calculation of $R^d(X)$ 251 | 5.4 | De Rham Groups | 168 |
| 5.6 Star-shaped Sets and Poincaré's Lemma1765.7 De Rham Groups of Spheres and Projective Spaces1785.8 De Rham Groups of Tori1825.9 Exercises184Chapter 6. Integration of Differential Forms6.1 Integrating Forms of Maximal Degree1896.2 Stokes' Theorem1956.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2196.8 Volume of Tubes III2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 5.5 | Lie Derivatives | 172 |
| 5.7 De Rham Groups of Spheres and Projective Spaces1785.8 De Rham Groups of Tori1825.9 Exercises184Chapter 6. Integration of Differential Forms6.1 Integrating Forms of Maximal Degree1896.2 Stokes' Theorem1956.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2196.8 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 5.6 | Star-shaped Sets and Poincaré's Lemma | 176 |
| 5.8 De Rham Groups of Tori1825.9 Exercises184Chapter 6. Integration of Differential Forms6.1 Integrating Forms of Maximal Degree1896.2 Stokes' Theorem1956.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2196.8 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 5.7 | De Rham Groups of Spheres and Projective Spaces | 178 |
| 5.9 Exercises184Chapter 6. Integration of Differential Forms1886.1 Integrating Forms of Maximal Degree1896.2 Stokes' Theorem1956.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2196.8 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 5.8 | De Rham Groups of Tori | 182 |
| Chapter 6. Integration of Differential Forms1886.1 Integrating Forms of Maximal Degree1896.2 Stokes' Theorem1956.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2196.8 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 5.9 | Exercises | 184 |
| 6.1 Integrating Forms of Maximal Degree1896.2 Stokes' Theorem1956.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2196.8 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas7.2 Calculation of $R^d(X)$ 251 | Chapte | er 6. Integration of Differential Forms | 188 |
| 6.2Stokes' Theorem1956.3First Applications of Stokes' Theorem1996.4Canonical Volume Forms2036.5Volume of a Submanifold of Euclidean Space2076.6Canonical Density on a Submanifold of Euclidean Space2146.7Volume of Tubes I2196.8Volume of Tubes II2276.9Volume of Tubes III2336.10Exercises238Chapter 7. Degree Theory7.1Preliminary Lemmas2457.2Calculation of $R^d(X)$ 251 | 6.1 | Integrating Forms of Maximal Degree | 189 |
| 6.3 First Applications of Stokes' Theorem1996.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2196.8 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 6.2 | Stokes' Theorem | 195 |
| 6.4 Canonical Volume Forms2036.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2196.8 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 6.3 | First Applications of Stokes' Theorem | 199 |
| 6.5 Volume of a Submanifold of Euclidean Space2076.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2196.8 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory7.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 6.4 | Canonical Volume Forms | 203 |
| 6.6 Canonical Density on a Submanifold of Euclidean Space2146.7 Volume of Tubes I2196.8 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 6.5 | Volume of a Submanifold of Euclidean Space | 207 |
| 6.7 Volume of Tubes I2196.8 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2442447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 6.6 | Canonical Density on a Submanifold of Euclidean Space | 214 |
| 6.8 Volume of Tubes II2276.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2442447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 6.7 | Volume of Tubes I | 219 |
| 6.9 Volume of Tubes III2336.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 6.8 | Volume of Tubes II | 227 |
| 6.10 Exercises238Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 6.9 | Volume of Tubes III | 233 |
| Chapter 7. Degree Theory2447.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | 6.10 | Exercises | 238 |
| 7.1 Preliminary Lemmas2457.2 Calculation of $R^d(X)$ 251 | Chapte | er 7. Degree Theory | 244 |
| 7.2 Calculation of $R^d(X)$ 251 | 7.1 | Preliminary Lemmas | 245 |
| | 7.2 | Calculation of $R^d(X)$ | 251 |

| Contents |
|----------|
|----------|

| 7.3 | The Degree of a Map | 253 |
|--------------|--|-----|
| 7.4 | Invariance under Homotopy. Applications | 256 |
| 7.5 | Volume of Tubes and the Gauss-Bonnet Formula | 262 |
| 7.6 | Self-Maps of the Circle | 267 |
| 7.7 | Index of Vector Fields on Abstract Manifolds | 270 |
| 7.8 | Exercises | 273 |
| Chapte | er 8. Curves: The Local Theory | 277 |
| 8.0 | Introduction | 278 |
| 8.1 | Definitions | 279 |
| 8.2 | Affine Invariants: Tangent, Osculating Plan, Concavity | 283 |
| 8.3 | Arclength | 288 |
| 8.4 | Curvature | 290 |
| 8.5 | Signed Curvature of a Plane Curve | 294 |
| 8.6 | Torsion of Three-Dimensional Curves | 297 |
| 8.7 | Exercises | 304 |
| Chapte | er 9. Plane Curves: The Global Theory | 312 |
| 9.1 | Definitions | 313 |
| 9.2 | Jordan's Theorem | 316 |
| 9.3 | The Isoperimetric Inequality | 322 |
| 9.4 | The Turning Number | 324 |
| 9.5 | The Turning Tangent Theorem | 328 |
| 9.6 | Global Convexity | 331 |
| 9.7 | The Four-Vertex Theorem | 334 |
| 9.8 | The Fabricius–Bjerre–Halpern Formula | 338 |
| 9.9 | Exercises | 344 |
| Chapte | er 10. A Guide to the Local Theory of Surfaces in \mathbb{R}^3 | 346 |
| 10.1 | Definitions | 348 |
| 10.2 | Examples | 348 |
| 10.3 | The Two Fundamental Forms | 369 |
| 10.4 | What the First Fundamental Form Is Good For | 371 |
| 10.5 | Gaussian Curvature | 382 |
| 10.6 | What the Second Fundamental Form Is Good For | 388 |
| 10.7 | Links Between the two Fundamental Forms | 401 |
| 10.8 | A Word about Hypersurfaces in \mathbb{R}^{n+1} | 402 |
| Chapte | er 11. A Guide to the Global Theory of Surfaces | 403 |
| Part I: | Intrinsic Surfaces | |
| 11 .1 | Shortest Paths | 405 |
| 11.2 | Surfaces of Constant Curvature | 407 |
| 11.3 | The Two Variation Formulas | 409 |
| 11.4 | Shortest Paths and the Injectivity Radius | 410 |

| 11.5 | Manifolds with Curvature Bounded Below | 414 |
|---------|--|-----|
| 11.6 | Manifolds with Curvature Bounded Above | 416 |
| 11.7 | The Gauss-Bonnet and Hopf Formulas | 417 |
| 11.8 | The Isoperimetric Inequality on Surfaces | 419 |
| 11.9 | Closed Geodesics and Isosystolic Inequalities | 420 |
| 11.10 | Surfaces All of Whose Geodesics Are Closed | 422 |
| 11.11 | Transition: Embedding and Immersion Problems | 423 |
| Part II | : Surfaces in \mathbb{R}^3 | |
| 11.12 | Surfaces of Zero Curvature | 425 |
| 11.13 | Surfaces of Non-Negative Curvature | 425 |
| 11.14 | Uniqueness and Rigidity Results | 427 |
| 11.15 | Surfaces of Negative Curvature | 428 |
| 11.16 | Minimal Surfaces | 429 |
| 11.17 | Surfaces of Constant Mean Curvature, or Soap Bubbles | 431 |
| 11.18 | Weingarten Surfaces | 433 |
| 11.19 | Envelopes of Families of Planes | 435 |
| 11.20 | Isoperimetric Inequalities for Surfaces | 437 |
| 11.21 | A Pot-pourri of Characteristic Properties | 438 |
| Biblio | graphy | 443 |
| Index | of Symbols and Notations | 453 |
| Index | | 456 |