

An Introduction to Wavelets

CHARLES K. CHUI

*Department of Mathematics
Texas A&M University
College Station, Texas*



ACADEMIC PRESS, INC.

Harcourt Brace Jovanovich, Publishers

Boston San Diego New York

London Sydney Tokyo Toronto

Contents

Preface	ix
1. An Overview	1
1.1. From Fourier analysis to wavelet analysis	1
1.2. The integral wavelet transform and time-frequency analysis	6
1.3. Inversion formulas and duals	9
1.4. Classification of wavelets	13
1.5. Multiresolution analysis, splines, and wavelets	16
1.6. Wavelet decompositions and reconstructions	18
2. Fourier Analysis	23
2.1. Fourier and inverse Fourier transforms	23
2.2. Continuous-time convolution and the delta function	27
2.3. Fourier transform of square-integrable functions	32
2.4. Fourier series	36
2.5. Basic convergence theory and Poisson's summation formula	43
3. Wavelet Transforms and Time-Frequency Analysis	49
3.1. The Gabor transform	50
3.2. Short-time Fourier transforms and the Uncertainty Principle	54
3.3. The integral wavelet transform	60
3.4. Dyadic wavelets and inversions	64
3.5. Frames	68
3.6. Wavelet series	74
4. Cardinal Spline Analysis	81
4.1. Cardinal spline spaces	81
4.2. <i>B</i> -splines and their basic properties	85
4.3. The two-scale relation and an interpolatory graphical display algorithm	90
4.4. <i>B</i> -net representations and computation of cardinal splines	95
4.5. Construction of spline approximation formulas	100
4.6. Construction of spline interpolation formulas	109

5. Scaling Functions and Wavelets	119
5.1. Multiresolution analysis	120
5.2. Scaling functions with finite two-scale relations	128
5.3. Direct-sum decompositions of $L^2(\mathbb{R})$	140
5.4. Wavelets and their duals	146
5.5. Linear-phase filtering	159
5.6. Compactly supported wavelets	168
6. Cardinal Spline-Wavelets	177
6.1. Interpolatory spline-wavelets	177
6.2. Compactly supported spline-wavelets	182
6.3. Computation of cardinal spline-wavelets	187
6.4. Euler-Frobenius polynomials	195
6.5. Error analysis in spline-wavelet decomposition	199
6.6. Total positivity, complete oscillation, zero-crossings	207
7. Orthogonal Wavelets and Wavelet Packets	215
7.1. Examples of orthogonal wavelets	215
7.2. Identification of orthogonal two-scale symbols	220
7.3. Construction of compactly supported orthogonal wavelets	229
7.4. Orthogonal wavelet packets	236
7.5. Orthogonal decomposition of wavelet series	240
Notes	245
References	251
Subject Index	257
Appendix	265