## Lecture Notes in Economics and Mathematical Systems

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Anders Borglin Hans Keiding



Optimality in Infinite Horizon Economies



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