

INTERNATIONAL CENTRE FOR MECHANICAL SCIENCES

COURSES AND LECTURES - No. 220



APPLICATION OF  
INTEGRAL TRANSFORMS  
IN THE THEORY OF ELASTICITY

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