



HANDBOOK OF MATHEMATICAL ECONOMICS

VOLUME IV

Edited by

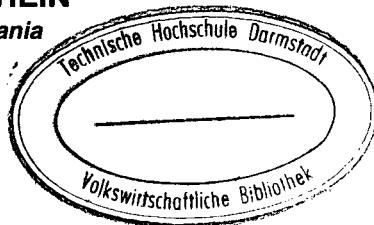
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