

# Numerical Recipes

in Fortran 90

The Art of *Parallel* Scientific Computing

Second Edition

Volume 2 of  
Fortran Numerical Recipes

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