

Functional equations in several variables

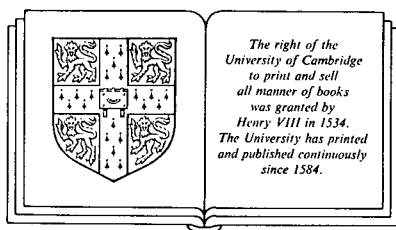
with applications to mathematics, information theory
and to the natural and social sciences

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