Statistical Techniques in Simulation

(IN TWO PARTS)

JACK P. C. KLEIJNEN

Katholieke Hogeschool Tilburg, The Netherlands

PART I

Te F	ch A(nisc CHB	he ER	Un El(ivera CH II	NFC	Der DR⊮	met	ædt K
B		B	L		0	Т	Н	Ε	К
Inven	lar-l	Nr.:	h	05	-00	200	37		
Sachg	jebi	eis:							
Stand	ort:	-							

MARCEL DEKKER, INC.

New York

CONTENTS OF PART I

	iv
DEDAGE	~~
PREFACE	XI
ACKNOWLEDGMENTS	xv
Chapter I FUNDAMENTALS OF SIMULATION	1
I.l Introduction and Summary	1
I.2 Systems	1
I.3 Models	3
I.4 Solution Methods	5
I.5 Monte Carlo	6
I.6 Simulation	12
I.7 Literature	28
Glossary	29
Appendix I.1 Monte Carlo Estimation of the Value	
of an Integral	30
Appendix I.2 An Example of Distribution Sampling	32
Appendix I.3 Simulation of Maintenance Strategies	35
Appendix I.4 Lewis-Learmonth Random Number	1
Generator	40
Appendix I.5 Sampling Two Correlated Variables	46
Exercises	49
Notes	49
References	51
Chapter II THE STATISTICAL ASPECTS OF SIMULATION	63
II.1 Introduction and Summary	63
II.2 Formulation of the Problem	65
II.3 Input Data Analysis	66
II.4 The Model and Computer Program	69
II.5 Validation	73

CONTENTS

II.6 Experimental Design 76 II.7 Analysis of Several Systems 80 II.8 Sample Size 82 II.9 Analysis of Steady-State Output and Runlength 87 II.10 Miscellaneous Statistical Aspects 90 II.11 Literature 92 Exercises 93 Notes 94 References 96 Chapter III VARIANCE REDUCTION TECHNIQUES 105 III.1 Introduction and Summary 105 III.2 Stratified Sampling 110 III.3 Selective Sampling or the Fixed Sequence Method 133 III.4 Control Variates or Regression Sampling 138 III.5 Importance Sampling 164 III.6 Antithetic Variates 186 III.7 Common Random Numbers 200 III.8 Joint Application of Antithetic Variates and Common Random Numbers 207 III.9 Summary and Conclusions 239 Appendix III.1 The Jackknife Statistic 240 Appendix III.2 Stratification After Sampling with Combinatio				
II.7 Analysis of Several Systems 80 II.8 Sample Size 82 II.9 Analysis of Steady-State Output and Runlength 87 II.10 Miscellaneous Statistical Aspects 90 II.11 Literature 92 Exercises 93 Notes 94 References 96 Chapter III VARIANCE REDUCTION TECHNIQUES 105 III.1 Introduction and Summary 105 III.2 Stratified Sampling 110 III.3 Selective Sampling or the Fixed Sequence Method 133 III.4 Control Variates or Regression Sampling 136 III.5 Importance Sampling 164 III.6 Antithetic Variates 106 III.8 Joint Application of Antithetic Variates and Common Random Numbers 200 III.9 Summary and Conclusions 238 III.10 Literature 239 Appendix III.1 The Jackknife Statistic 240 Appendix III.2 The Mean Square Error 244 Appendix III.3 The Mean Square Error 244 Appendix III.4 The Estimated Variance in Stratification after Sampling 247 Appendix III.5 An Example of the Generation of Random Numbers in Stratified Sampling 246	11.6	Experimenta	l Design	76
II.8 Sample Size 82 II.9 Analysis of Steady-State Output and Runlength 87 II.10 Miscellaneous Statistical Aspects 90 II.11 Literature 92 Exercises 93 Notes 94 References 96 Chapter III VARIANCE REDUCTION TECHNIQUES 105 III.1 Introduction and Summary 105 III.2 Stratified Sampling 110 III.3 Selective Sampling or the Fixed Sequence Method 133 III.4 Control Variates or Regression Sampling 138 III.5 Importance Sampling 164 III.6 Antithetic Variates 200 III.8 Joint Application of Antithetic Variates and Common Random Numbers 207 III.9 Summary and Conclusions 238 III.10 Literature 239 Appendix III.1 The Jackknife Statistic 240 Appendix III.2 The Mean Square Error 244 Appendix III.4 The Estimated Variance in Stratification of Random Numbers in Stratified Sampling 247 Appendix III.5 An Example of the Generation of Random Numbers in Stratified Sampling 248 Appendix III.6 Attrates Approximation for the Optimal Density Function in the Importance Sampling Density Function in the Im	II.7	Analysis of	Several Systems	80
II.9 Analysis of Steady-State Output and Runlength 87 II.10 Miscellaneous Statistical Aspects	II.8	Sample Size	••••••••••••••••	82
II.10 Miscellaneous Statistical Aspects 90 II.11 Literature 92 Exercises 93 Notes 94 References 96 Chapter III VARIANCE REDUCTION TECHNIQUES 105 III.1 Introduction and Summary 105 III.2 Stratified Sampling 110 III.3 Selective Sampling or the Fixed Sequence Method 133 III.4 Control Variates or Regression Sampling 138 III.5 Importance Sampling 164 III.6 Antithetic Variates 186 III.7 Common Random Numbers 200 III.8 Joint Application of Antithetic Variates and Common Random Numbers 207 III.9 Summary and Conclusions 238 III.10 Literature 239 Appendix III.1 The Jackknife Statistic 240 Appendix III.2 Stratification After Sampling with Combination of Strata 242 Appendix III.3 The Mean Square Error 244 Appendix III.5 An Example of the Generation of Random Numbers in Stratified Sampling 247 Appendix III.5 An Example of the Generation of Random Numbers in Stratified Sampling 247 Appendix III.7 The Covariances Among Control Variate Estimators 248	II.9	Analysis of	Steady-State Output and Runlength	87
II.11 Literature 92 Exercises 93 Notes 94 References 96 Chapter III VARIANCE REDUCTION TECHNIQUES 105 III.1 Introduction and Summary 105 III.2 Stratified Sampling 110 III.3 Selective Sampling or the Fixed Sequence Method 133 III.4 Control Variates or Regression Sampling 138 III.5 Importance Sampling 164 III.6 Antithetic Variates 186 III.7 Common Random Numbers 200 III.8 Joint Application of Antithetic Variates and Common Random Numbers 207 III.9 Summary and Conclusions 238 III.10 Literature 239 Appendix III.1 The Jackknife Statistic 240 Appendix III.2 Stratification After Sampling with Combination of Strata 242 Appendix III.3 The Mean Square Error 244 Appendix III.5 An Example of the Generation of Random Numbers in Stratified Sampling 247 Appendix III.6 Expected Value of a Control Variate with Stochastic <u>m</u> 248 Appendix III.7 The Covariances Among Control Variate Estimators 249 Appendix III.8 Gamma Approximation for the Optimal Density Function in the Importance	II.10	Miscellaneo	us Statistical Aspects	90
Exercises 93 Notes 94 References 96 Chapter III VARIANCE REDUCTION TECHNIQUES 105 III.1 Introduction and Summary 105 III.2 Stratified Sampling 110 III.3 Selective Sampling or the Fixed Sequence Method 133 III.4 Control Variates or Regression Sampling 136 III.5 Importance Sampling 164 III.6 Antithetic Variates 186 III.7 Common Random Numbers 200 III.8 Joint Application of Antithetic Variates and Common Random Numbers 207 III.9 Summary and Conclusions 238 III.10 Literature 239 Appendix III.1 The Jackknife Statistic 240 Appendix III.2 Stratification After Sampling with Combination of Strata 242 Appendix III.3 The Mean Square Error 244 Appendix III.4 The Estimated Variance in Stratifi- cation after Sampling 247 Appendix III.5 An Example of the Generation of Random Numbers in Stratified Sampling 247 Appendix III.6 Expected Value of a Control Variate with Stochastic <u>m</u> 248 Appendix III.7 The Covariances Among Control Variate Estimators 249 Appendix III.8 Gamma Approximation	II.11	Literature	•••••••••••••••••••••••••••••••••••••••	92
Notes 94 References 96 Chapter III VARIANCE REDUCTION TECHNIQUES 105 III.1 Introduction and Summary 105 III.2 Stratified Sampling 110 III.3 Selective Sampling or the Fixed Sequence Method 133 III.4 Control Variates or Regression Sampling 138 III.5 Importance Sampling 164 III.6 Antithetic Variates 186 III.7 Common Random Numbers 200 III.8 Joint Application of Antithetic Variates and Common Random Numbers 207 III.9 Summary and Conclusions 238 III.10 Literature 239 Appendix III.1 The Jackknife Statistic 240 Appendix III.2 Stratification After Sampling with Combination of Strata 244 Appendix III.3 The Mean Square Error 244 Appendix III.4 The Estimated Variance in Stratifi- cation after Sampling 247 Appendix III.5 An Example of the Generation of Random Numbers in Stratified Sampling 248 Appendix III.7 The Covariances Among Control Variate Estimators 249 Appendix III.7 The Covariances Among Control Variate Estimators 249 Appendix III.8 Gamma Approximation for the Optimal Density Function in the Importance 24	Exe	rcises	•••••••••••••••••	93
References 96 Chapter III VARIANCE REDUCTION TECHNIQUES 105 III.1 Introduction and Summary 105 III.2 Stratified Sampling 110 III.3 Selective Sampling or the Fixed Sequence Method 133 III.4 Control Variates or Regression Sampling 138 III.5 Importance Sampling 164 III.6 Antithetic Variates 186 III.7 Common Random Numbers 200 III.8 Joint Application of Antithetic Variates and Common Random Numbers 207 III.9 Summary and Conclusions 238 III.10 Literature 239 Appendix III.1 The Jackknife Statistic 240 Appendix III.2 Stratification After Sampling with Combination of Strata 242 Appendix III.3 The Mean Square Error 244 Appendix III.5 An Example of the Generation of Random Numbers in Stratified Sampling 247 Appendix III.6 Expected Value of a Control Variate with Stochastic m 248 Appendix III.7 The Covariances Among Control Variate Estimators 249 Appendix III.8 Gamma Approximation for the Optimal Density Function in the Importance Sampling Example 249	Note	es		94
Chapter III VARIANCE REDUCTION TECHNIQUES 105 III.1 Introduction and Summary 105 III.2 Stratified Sampling 110 III.3 Selective Sampling or the Fixed Sequence Method 133 III.4 Control Variates or Regression Sampling 138 III.5 Importance Sampling 164 III.6 Antithetic Variates 186 III.7 Common Random Numbers 200 III.8 Joint Application of Antithetic Variates and Common Random Numbers 207 III.9 Summary and Conclusions 238 III.10 Literature 239 Appendix III.1 The Jackknife Statistic 240 Appendix III.2 Stratification After Sampling with Combination of Strata 242 Appendix III.3 The Mean Square Error 244 Appendix III.4 The Estimated Variance in Stratifi- cation after Sampling 247 Appendix III.5 An Example of the Generation of Random Numbers in Stratified Sampling 247 Appendix III.6 Expected Value of a Control Variate with Stochastic m 248 Appendix III.7 The Covariances Among Control Variate Estimators 249 Appendix III.8 Gamma Approximation for the Optimal Density Function in the Importance Sampling Example 249	Refe	erences		96
III.1Introduction and Summary105III.2Stratified Sampling110III.3Selective Sampling or the Fixed Sequence Method133III.4Control Variates or Regression Sampling138III.5Importance Sampling164III.6Antithetic Variates186III.7Common Random Numbers200III.8Joint Application of Antithetic Variates and Common Random Numbers207III.9Summary and Conclusions238III.10Literature239Appendix III.1The Jackknife Statistic240Appendix III.2Stratification After Sampling with Combination of Strata242Appendix III.3The Mean Square Error244Appendix III.5An Example of the Generation of Random Numbers in Stratified Sampling247Appendix III.6Expected Value of a Control Variate with Stochastic m248Appendix III.7The Covariances Among Control Variate Estimators249Appendix III.8Gamma Approximation for the Optimal Density Function in the Importance Sampling Example251	Chapter :	III VARIANC	E REDUCTION TECHNIQUES	105
III.2Stratified Sampling110III.3Selective Sampling or the Fixed Sequence Method133III.4Control Variates or Regression Sampling138III.5Importance Sampling164III.6Antithetic Variates186III.7Common Random Numbers200III.8Joint Application of Antithetic Variates and Common Random Numbers207III.9Summary and Conclusions238III.10Literature239Appendix III.1The Jackknife Statistic240Appendix III.2Stratification After Sampling with Combination of Strata242Appendix III.3The Mean Square Error244Appendix III.5An Example of the Generation of Random Numbers in Stratified Sampling247Appendix III.6Expected Value of a Control Variate with Stochastic m248Appendix III.7The Covariances Among Control Variate Estimators249Appendix III.8Gamma Approximation for the Optimal Density Function in the Importance Sampling Example251	III.l	Introducti	on and Summary	105
III.3 Selective Sampling or the Fixed Sequence Method133III.4 Control Variates or Regression Sampling138III.5 Importance Sampling164III.6 Antithetic Variates186III.7 Common Random Numbers	III . 2	Stratified	Sampling	110
III.4 Control Variates or Regression Sampling138III.5 Importance Sampling164III.6 Antithetic Variates186III.7 Common Random Numbers200III.8 Joint Application of Antithetic Variates and Common Random Numbers207III.9 Summary and Conclusions238III.10 Literature239Appendix III.1 The Jackknife Statistic240Appendix III.2 Stratification After Sampling with Combination of Strata242Appendix III.3 The Mean Square Error244Appendix III.5 An Example of the Generation of Random Numbers in Stratified Sampling247Appendix III.6 Expected Value of a Control Variate with Stochastic m248Appendix III.7 The Covariances Among Control Variate Estimators249Appendix III.8 Gamma Approximation for the Optimal Density Function in the Importance Sampling Example251	III.3	Selective	Sampling or the Fixed Sequence Method	133
III.5 Importance Sampling164III.6 Antithetic Variates186III.7 Common Random Numbers200III.8 Joint Application of Antithetic Variates and Common Random Numbers207III.9 Summary and Conclusions238III.10 Literature239Appendix III.1 The Jackknife Statistic240Appendix III.2 Stratification After Sampling with Combination of Strata242Appendix III.3 The Mean Square Error244Appendix III.4 The Estimated Variance in Stratific cation after Sampling246Appendix III.5 An Example of the Generation of Random Numbers in Stratified Sampling247Appendix III.6 Expected Value of a Control Variate with Stochastic m248Appendix III.7 The Covariances Among Control Variate Estimators249Appendix III.8 Gamma Approximation for the Optimal Density Function in the Importance Sampling Example251	III.4	Control Va	riates or Regression Sampling	138
III.6Antithetic Variates186III.7Common Random Numbers200III.8Joint Application of Antithetic Variates and Common Random Numbers207III.9Summary and Conclusions238III.10Literature239Appendix III.1The Jackknife Statistic240Appendix III.2Stratification After Sampling with Combination of Strata242Appendix III.3The Mean Square Error244Appendix III.4The Estimated Variance in Stratific cation after Sampling247Appendix III.5An Example of the Generation of Random Numbers in Stratified Sampling247Appendix III.6Expected Value of a Control Variate with Stochastic m248Appendix III.7The Covariances Among Control Variate Estimators249Appendix III.8Gamma Approximation for the Optimal Density Function in the Importance Sampling Example251	III . 5	Importance	Sampling	164
III.7Common Random Numbers200III.8Joint Application of Antithetic Variates and Common Random Numbers207III.9Summary and Conclusions238III.10Literature239Appendix III.1The Jackknife Statistic240Appendix III.2Stratification After Sampling with Combination of Strata242Appendix III.3The Mean Square Error244Appendix III.4The Estimated Variance in Stratification after Sampling246Appendix III.5An Example of the Generation of Random Numbers in Stratified Sampling247Appendix III.6Expected Value of a Control Variate with Stochastic m248Appendix III.7The Covariances Among Control Variate Estimators249Appendix III.8Gamma Approximation for the Optimal Density Function in the Importance Sampling Example251	III.6	Antithetic	Variates	186
III.8 Joint Application of Antithetic Variates and Common Random Numbers	III.7	Common Ran	dom Numbers	200
III.9Summary and Conclusions238III.10Literature239Appendix III.1The Jackknife Statistic240Appendix III.2Stratification After Sampling with Combination of Strata242Appendix III.3The Mean Square Error244Appendix III.4The Estimated Variance in Stratification after Sampling246Appendix III.5An Example of the Generation of Random Numbers in Stratified Sampling247Appendix III.6Expected Value of a Control Variate with Stochastic m248Appendix III.7The Covariances Among Control 	III.8	Joint Appl Common Ran	ication of Antithetic Variates and dom Numbers	207
III.10 Literature 239 Appendix III.1 The Jackknife Statistic 240 Appendix III.2 Stratification After Sampling with Combination of Strata 242 Appendix III.3 The Mean Square Error 244 Appendix III.4 The Estimated Variance in Stratification after Sampling 246 Appendix III.5 An Example of the Generation of Random Numbers in Stratified Sampling 247 Appendix III.6 Expected Value of a Control Variate with Stochastic m 248 Appendix III.7 The Covariances Among Control Variate Estimators 249 Appendix III.8 Gamma Approximation for the Optimal Density Function in the Importance Sampling Example 251	III.9	Summary an	d Conclusions	238
Appendix III.1The Jackknife Statistic	III.l	0 Literature	••••••••••••••	239
Appendix III.2Stratification After Sampling with Combination of Strata242Appendix III.3The Mean Square Error244Appendix III.4The Estimated Variance in Stratification after Sampling246Appendix III.5An Example of the Generation of Random Numbers in Stratified Sampling247Appendix III.6Expected Value of a Control Variate with Stochastic m248Appendix III.7The Covariances Among Control Variate Estimators249Appendix III.8Gamma Approximation for the Optimal Density Function in the Importance Sampling Example251	Appe	endix III.l	The Jackknife Statistic	240
Appendix III.3 The Mean Square Error	Appe	endix III.2	Stratification After Sampling with Combination of Strata	242
Appendix III.4 The Estimated Variance in Stratification after Sampling	Appe	endix III.3	The Mean Square Error	244
Appendix III.5 An Example of the Generation of Random Numbers in Stratified Sampling	App	endix III.4	The Estimated Variance in Stratifi- cation after Sampling	246
Appendix III.6 Expected Value of a Control Variate with Stochastic m 248 Appendix III.7 The Covariances Among Control Variate Estimators 249 Appendix III.8 Gamma Approximation for the Optimal Density Function in the Importance Sampling Example 251	Appe	endix III.5	An Example of the Generation of Random Numbers in Stratified Sampling	247
Appendix III.7The Covariances Among Control Variate Estimators249Appendix III.8Gamma Approximation for the Optimal Density Function in the Importance Sampling Example251	Appe	endix III.6	Expected Value of a Control Variate with Stochastic \underline{m}	248
Appendix III.8 Gamma Approximation for the Optimal Density Function in the Importance Sampling Example	Appe	endix III.7	The Covariances Among Control Variate Estimators	249
	Арре	endix III.8	Gamma Approximation for the Optimal Density Function in the Importance Sampling Example	251

I

ł

I

ł

Appendix I	11.9	The Variance of the Importance Sampling Estimator $g_{z}^{*}(\underline{x})$	253
Appendix I	11.10	Generating Antithetic Random Numbers from a Multiplicative Congruential Relation	254
Appendix I	11.11	The Variance of the Estimated Difference between the Responses of Two Systems	2 56
Appendix I	11.12	The Signs of the Coefficients a ₁ and a ₂	258
Appendix I	II.13	Optimum Computer Time Allocation	259
Exercises	• • • • •		263
Notes			265
References	• • • •		275

vii