

# Analysis and Control of Dynamic Economic Systems

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**John Wiley & Sons**

**New York • London • Sydney • Toronto**

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## PART I. ANALYSIS OF DYNAMIC ECONOMIC SYSTEMS

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