

Gaussian Markov Random Fields

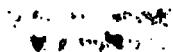
Theory and Applications

Håvard Rue
Leonhard Held



Chapman & Hall/CRC
Taylor & Francis Group

Boca Raton London New York Singapore



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