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Postoptimal Analyses, Parametric Programming, and Related Topics

Degeneracy, Multicriteria Decision Making, Redundancy

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