

Logic of Choice and Economic Theory

S. N. AFRIAT

CLARENDON PRESS · OXFORD
1987

Contents

Introduction	1
Part I	
Choice and the Optimum	
<i>Chapter I.1 About Choice</i>	13
1.1 Forming a choice	14
1.2 Demand as choice	16
1.3 Choice functions	20
1.4 The Optimum	21
1.5 Optimal choice functions	23
1.6 Revealed choice and preference	24
1.7 Dinners and Battles	27
1.8 The Maximum Doctrine	29
<i>Chapter I.2 Processes of Choice</i>	32
2.1 Search and elimination	34
2.2 Binary choice	36
2.3 Preference	39
2.4 Revealed preference	42
2.5 Revealed contradictions	44
<i>Chapter I.3 Democratic Choice</i>	47
3.1 Axiomatics of voting	50
3.2 Binary elections	53
3.3 The Voting Paradox	55
3.4 Electing an order	57
3.5 Democratic impartiality	59
3.6 Irrelevance Principle	61
3.7 Hansson's Group Indifference theorem	63
3.8 Positive Association	66
3.9 Arrow's Group Dictatorship theorem	68
<i>Chapter I.4 Budget Allocation and Priority</i>	71
4.1 The Allocation Rule	71
4.2 Marginal utility of money	73
4.3 Many goods	73
4.4 Priority	74

4.5 Law of Returns	74
4.6 Adjustment and Priority Rules	76
<i>Chapter I.5 Ramsey's Savings Rule</i>	78
5.1 Counting and discounting	79
5.2 Income disposition	80
5.3 Ramsey's problem	80
5.4 The Rule	81
5.5 Keynes's argument	83
5.6 Ramsey's argument	85
5.7 Time paths	85
5.8 A special case	87
5.9 Discounted future	87

Part II **Use and Exchange**

<i>Chapter II.1 Utility Hypothesis</i>	93
1.1 Needs of measurement	97
1.2 Common practice, and Fleetwood	100
1.3 Parallels in theory	103
1.4 Revealed preference	106
1.5 Classical case	109
<i>Chapter II.2 Algebra of Revealed Preference</i>	112
2.1 Demand and utility	114
2.2 Consistency conditions	116
2.3 Order logic	118
2.4 Revealed preference revealed	120
2.5 Finite Houthakker test	123
2.6 Constructive solution	126
2.7 Multipliers and levels	129
<i>Chapter II.3 Combinatorics of Demand</i>	132
3.1 General consistency	133
3.2 Cross-costs	136
3.3 Classical consistency	139
3.4 Multiplier-level solutions	141
3.5 Utility cost	143
3.6 Classical limits	145
<i>Chapter II.4 Separable Utility</i>	148
4.1 Models of separability	148
4.2 Separability tests	149
4.3 Constructions	150
4.4 Utility dimension	151
4.5 Budget separability	153

<i>Chapter II.5 Direct and Indirect Utility</i>	156
5.1 Purchasing power	157
5.2 The indirect ‘integrability’ problem	158
5.3 Basic relations and properties	162
5.4 Adjoint of a relation	166
5.5 Adjoint of a function	169
5.6 Limit adjoints	170
<i>Chapter II.6 Efficiency and Inefficiency</i>	173
6.1 Consumer inefficiency	176
6.2 Attainable efficiencies	177
6.3 Utility approximation	179
6.4 Production efficiency	180
 Part III	
The Cost of Living	
<i>Chapter III.1 Price and Quantity Levels</i>	185
1.1 Price-quantity duality	186
1.2 Dual function examples	188
1.3 Price and quantity levels	191
1.4 Limits of indeterminacy	193
<i>Chapter III.2 The True Index</i>	195
2.1 The cost of living	195
2.2 The price index	196
2.3 Formulae, and Fisher’s Tests	196
2.4 The Paasche-Laspeyres interval	197
2.5 Existence test	199
2.6 Theory and practice	200
2.7 Many periods	202
2.8 Price levels	205
2.9 Fisher’s formula	209
<i>Chapter III.3 Fisher and Byushgens</i>	211
3.1 Byushgens’s theorem	213
3.2 The existence question	214
3.3 Purchasing power correspondence	217
3.4 Many-period generalization	219
<i>Chapter III.4 The Four-point Formula</i>	222
4.1 Median multipliers and levels	227
4.2 Centre locus	229
4.3 Linear purchasing power	232
4.4 Critical locations	234
4.5 Elliptical case	236

4.6 Hyperbolic case	238
4.7 Parabolic limits	240
4.8 Demonstration: Fisher's data	241
<i>Chapter III.5 Wald's 'New Formula'</i>	246
5.1 Linear expansions	247
5.2 Revealed purchasing power	249
5.3 The critical points	250
5.4 Marginal price indices, and limits	252
Part IV	
Logic of Price	
<i>Chapter IV.1 Opportunity Models</i>	257
1.1 The production function	258
1.2 General input-output	259
1.3 Von Neumann's activity system	259
1.4 Axiomatics of Koopmans's system	262
<i>Chapter IV.2 Leontief's Input-Output</i>	266
2.1 Quesnay's Tableau économique	267
2.2 The Leontief Matrix	268
2.3 Production planning	269
2.4 Leontief and Zeno	270
2.5 Productive systems	271
2.6 Computer demonstration	273
<i>Chapter IV.3 The Market</i>	275
3.1 The logic of price	276
3.2 Supply and demand	277
3.3 Price formation	279
3.4 Market functions	281
3.5 Intercept and slope	282
3.6 Linear markets	283
3.7 Tâtonnement	285
3.8 Continuous markets	286
3.9 Uniqueness and stability	287
<i>Chapter IV.4 Sraffa's Prices</i>	289
4.1 Production for subsistence	291
4.2 Interdependence and stability	293
4.3 Production with a surplus	293
4.4 Joint production	295
4.5 Variable activity	296
4.6 Sraffa and Leontief	297
4.7 Sraffa and von Neumann	300

<i>Chapter IV.5 General Economic Equilibrium</i>	304
5.1 Crusoe's development	306
5.2 Edgeworth's box	308
5.3 Three senses of equilibrium	313
5.4 Feasible prices	316
5.5 A Cobb-Douglas world	318
5.6 The hyperbox	320
5.7 Production and profit	323
5.8 Lessons in Edgeworth's box	326
<i>Chapter IV.6 Von Neumann's Economic Model</i>	348
6.1 Activity system	349
6.2 Dual system	350
6.3 Irreducibility	351
6.4 Admissible factors	352
6.5 Factor bounds	353
6.6 Limit reciprocity	354
6.7 Von Neumann factor	355
6.8 Stretch algorithm	357
6.9 Matrix games	360
6.10 LP solution	361
6.11 Perron-Frobenius and von Neumann	362
6.12 BASIC program	364
6.13 Problems	368
6.14 Demonstrations	369

Part V Optimal Programming

<i>Chapter V.1 Optimal Programming</i>	377
1.1 Bounds, limits and maxima	380
1.2 Programming problem of a firm	383
1.3 Optimal programming theorem	385
1.4 Input-output	386
1.5 Output limit function	387
1.6 Support gradients and marginal values	388
1.7 Complementarity	390
1.8 Shadow price decentralization	391
1.9 Proof of the theorem	392
1.10 Lagrange multipliers	393
<i>Chapter V.2 Convex Programming</i>	401
2.1 Convexity	403
2.2 Programming convexity theorem	405
2.3 Slater's condition	407

2.4 Optimality theorem	407
2.5 Non-negative maxima	408
2.6 The Kuhn-Tucker conditions	409
<i>Chapter V.3 Linear Programming</i>	413
3.1 Linear inequalities	414
3.2 Separation theorems	421
3.3 Theorems of alternatives	425
3.4 Polyhedra and polytopes	429
3.5 LP duality	432
3.6 The pivot operation	434
3.7 The Simplex Algorithm	436
3.8 BASIC program	439
<i>Chapter V.4 Minimum Paths</i>	442
4.1 Connection costs	445
4.2 <i>Perpetuum mobile</i> impossible	446
4.3 The triangle inequality	447
4.4 Routes	448
4.5 Scales	451
4.6 Extension theorem	454
4.7 The LP formula	456
4.8 Flow argument	457
4.9 Elementary decomposition	459
4.10 Ford and Fulkerson	462
4.11 Shortest path algorithm in BASIC	464
<i>Chapter V.5 Distribution Matrices</i>	468
5.1 Equilibrium	469
5.2 Irreducibility	470
5.3 Powers and limits	472
5.4 Convergence	477
5.5 Periodic case	478
5.6 Computer graphics	480
Part VI	
General Mathematics	
<i>Chapter VI.1 Calculus of Propositions</i>	487
1.1 The Boolean laws	489
1.2 Normal forms	490
1.3 The conditional	491
1.4 Tautologies	492
1.5 Existential and universal quantifiers	494
1.6 Predicates and sets	495
1.7 Shao Yung's Program	497

<i>Contents</i>	xiii
Chapter VI.2 Algebra of Relations	499
2.1 Operations with relations	500
2.2 Classification of relations	503
2.3 Order and equivalence	506
2.4 Order refinement	509
2.5 Representation theorems	511
Chapter VI.3 Intersections and Fixed Points	519
3.1 Simplices and dissections	520
3.2 Sperner's lemma	520
3.3 The KKM lemma	522
3.4 Brouwer's fixed point theorem	523
3.5 Kakutani and von Neumann	524
3.6 The Nash equilibrium	526
3.7 Saddle points	528
3.8 BASIC Games	533
<i>Bibliography</i>	543
<i>Index</i>	569