

# Understanding Computation

*Tom Stuart*

**O'REILLY®**

Beijing • Cambridge • Farnham • Koln • Sebastopol • Tokyo

# Table of Contents

Preface	vii
1. Just Enough Ruby	1
Interactive Ruby Shell	1
Values	2
Basic Data	2
Data Structures	3
Procs	3
Control Flow	4
Objects and Methods	5
Classes and Modules	6
Miscellaneous Features	7
Local Variables and Assignment	7
String Interpolation	8
Inspecting Objects	8
Printing Strings	8
Variadic Methods	9
Blocks	9
Enumerable	10
Struct	11
Monkey Patching	12
Defining Constants	13
Removing Constants	13
Parti. Programs and Machines	
2. The Meaning of Programs	17
The Meaning of "Meaning"	18
Syntax	19
Operational Semantics	20
Small-Step Semantics	21

Big-Step Semantics	42
Denotational Semantics	49
Expressions	50
Statements	52
Applications	55
Formal Semantics in Practice	55
Formality	56
Finding Meaning	56
Alternatives	57
Implementing Parsers	58
3. The Simplest Computers	63
Deterministic Finite Automata	63
States, Rules, and Input	63
Output	64
Determinism	66
Simulation	66
Nondeterministic Finite Automata	69
Nondeterminism	70
Free Moves	76
Regular Expressions	79
Syntax	80
Semantics	83
Parsing	92
Equivalence	94
4. Just Add Power	105
Deterministic Pushdown Automata	108
Storage	108
Rules	110
Determinism	111
Simulation	112
Nondeterministic Pushdown Automata	118
Simulation	122
Nonequivalence	125
Parsing with Pushdown Automata	125
Lexical Analysis	126
Syntactic Analysis	128
Practicalities	132
How Much Power?	133
5. The Ultimate Machine	135
Deterministic Turing Machines	135

Storage	136
Rules	138
Determinism	141
Simulation	141
Nondeterministic Turing Machines	147
Maximum Power	148
Internal Storage	148
Subroutines	151
Multiple Tapes	153
Multidimensional Tape	154
General-Purpose Machines	154
Encoding	156
Simulation	157

## Part II. Computation and Computability

6. Programming with Nothing	161
Impersonating the Lambda Calculus	162
Working with Procs	163
The Problem	164
Numbers	166
Booleans	169
Predicates	172
Pairs	173
Numeric Operations	174
Lists	181
Strings	184
The Solution	186
Advanced Programming Techniques	192
Implementing the Lambda Calculus	197
Syntax	197
Semantics	199
Parsing	204
7. Universality Is Everywhere	207
Lambda Calculus	207
Partial Recursive Functions	210
SKI Combinator Calculus	215
Iota	224
Tag Systems	227
Cyclic Tag Systems	235
Conway's Game of Life	245

Rule 110	247
Wolfram's 2,3 Turing Machine	251
8. Impossible Programs	253
The Facts of Life	254
Universal Systems Can Perform Algorithms	254
Programs Can Stand In for Turing Machines	257
Code Is Data	258
Universal Systems Can Loop Forever	259
Programs Can Refer to Themselves	264
Decidability	269
The Halting Problem	271
Building a Halting Checker	271
It'll Never Work	274
Other Undecidable Problems	277
Depressing Implications	280
Why Does This Happen?	282
Coping with Uncomputability	283
9. Programming in Toyland	285
Abstract Interpretation	286
Route Planning	286
Abstraction: Multiplying Signs	287
Safety and Approximation: Adding Signs	290
Static Semantics	295
Implementation	296
Benefits and Limitations	303
Applications	305
Afterword	307
Index	309